Ideas about a fast running vane pump directly driven by a 0.37 kW, 4-pole asynchronous motor frame size 71

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1 Introduction

Water is a basic need of mankind but many places on earth suffer from water shortage. Even if the average yearly rainfall is enough, there will be dry periods when water has to be pumped. Many places on earth have no access to the electricity grid and then one has to use hand pumps or pumps driven by a motor aggregate or by renewable energy like sun or wind. As fuel prices are rising, the use of sun or wind energy becomes more and more attractive in developing countries. But because of the required investment costs, the price of energy generated by sun or wind is much higher than the price of energy supplied by the grid. So if sun or wind energy is used to power an electric pump, one has to use a pump with a high overall efficiency. This means that the ratio in between the obtained hydraulic energy in the water and the required electrical energy has to be as high as possible. However, a high efficiency should be realised at the lowest possible investment costs for the pump and the pump motor. Local manufacture of the pump may result in the lowest pump price.

The overall efficiency of an electric pump depends on the efficiency of the pump motor, the efficiency of the pump, the efficiency of the transmission in between motor and pump, the losses in the cables from the energy source to the pump motor and the hydraulic losses in the piping in between the well and the pump and in between the pump and the reservoir in which the water is pumped.

The highest motor efficiency is realised for a permanent magnet DC motor. An efficiency of 0.8 is possible for a small motor. However, these motors are rather expensive and may be difficult to obtain in developing countries. The nominal rotational speed is also higher than for a 4-pole asynchronous motor. The cable losses will be rather high as the nominal voltage is only 24 V. An alternative is to use an asynchronous 4-pole, 3-phase motor which runs at a nominal speed of about 1400 rpm. The motor efficiency η_m of such a motor with frame size 71 is about 0.65. The cable losses are low because a high 230/400 V, 3-phase voltage is chosen. It is estimated that the cable efficiency η_c for normal currents is 0.98. The highest pump efficiency η_p is possible for a piston pump. An efficiency of maximum 0.9 has been measured for single acting piston pumps. The efficiency of a vane pump will be lower because of internal water leakage. It is expected that a well designed vane pump will have an efficiency of 0.85 at the nominal rotational speed. The highest transmission efficiency is realised if there is no reducing transmission. So the vane pump will be mounted directly to the motor shaft. Then the transmission efficiency $\eta_t = 1$.

So the maximum total efficiency η_{tot} of the pump and an asynchronous motor including cables $\eta_{tot} = \eta_c * \eta_m * \eta_t * \eta_p = 0.98 * 0.65 * 1 * 0.85 = 0.54$. The hydraulic losses in the piping are minimised if the flow through the pipe is not fluctuating and if smooth pipes with sufficient inner diameter are used. The pipe resistance results in an extra dynamic head.

The main disadvantage of a single acting piston pump which is used in traditional water pumping windmills is that the torque and the flow varies sinusoidal during the upwards stroke and that it is zero during the downwards stroke. The peak torque is a factor π times the average torque. If an elastic element with a volume variation of about five times the stroke volume of the pump is added just after the pump, the flow coming out of this elastic element will be almost constant. This principle is used in the Solaflux solar pump which was tested in combination with the VIRYA-3B3 windmill already in 2003. This combination is described in chapter 5 of public report KD 490 (ref. 1).

Already in 2002 I did some research to a vane pump which is coupled to a windmill by means of a vertical shaft running in the tower centre. Report KD 109 (ref. 2) is the most recent report about the vane pump. However, this report is written in Dutch. The principle used for the pump can also be used for a pump driven by an electric motor. Chapters 2, 3 and a part of chapter 4 of report KD 109 are therefore translated into English.

2 Description of the vane pump

The vane pump has a rotor in which two sleeves are made under an angle of 90°. Both sleeves have the same depth. In two opposite sleeves, one strip is positioned. The strips jut out a little out of the rotor. Every strip has a cut-away in the centre to prevent that the strips touch each other in the area in which they normally move. The strips are rounded at the outside with a radius which is smaller than the smallest radius of the chamber made in the pump housing. The rotor is turning in a housing which has a chamber which is eccentric with respect to the rotor axis. The shape of the chamber is such that the strips have a minimal clearance in the housing. The depth of the chamber is identical to the height of the vanes. The housing has a cover at the left side. The rotor has two bearings at the right part of the housing which makes mounting of the strips easy. The cover is provided by two kidney shaped openings through which the water is sucked and pressed. As there must always be one vane in between the suction and the pressure side of the pump, the kidney shaped openings cover an area of 90°.

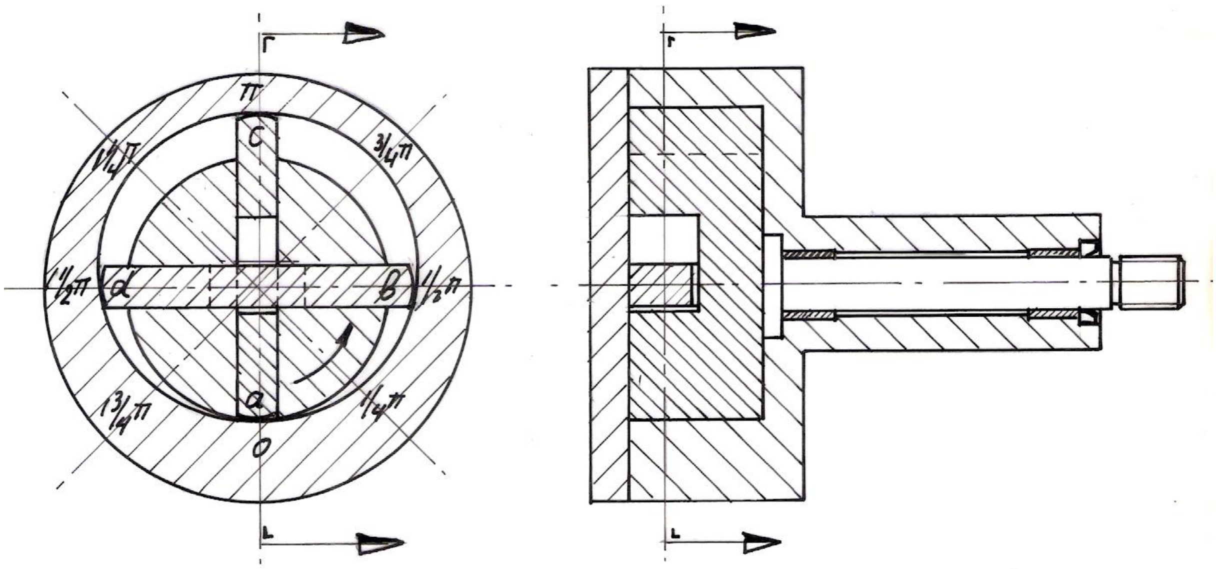

fig. 1 Vane pump

3 Determination of the shape of the chamber in the housing

In the left cross section of figure 1, one can distinguish two sectors which determine the functioning of the pump and the movement of the vanes. Each sector covers an area $\Delta\varphi = 180°$ or π radial. The right sector runs from 0 up to π rad. The left sector runs from π up to 2π rad. The two strips are forming four vanes which are numbered a, b, c and d. Vane a and c are a part of one strip. Vane b and d are a part of the other strip. The rotor is turning left hand if seen from the left side. The rotor is drawn in figure 1 for a position where vane a coincides with $\varphi = 0$.

The right sector, which runs from $\varphi = 0$ up to $\varphi = \pi$, is the area in which vane a is moving to the outside. The space in between the rotor and the chamber is increasing and therefore water is sucked in this area. The kidney shaped suction opening is lying in this sector and covers the area from ¼ π up to ¾ π.

The left sector, which runs from $\varphi = \pi$ up to $\varphi = 2\pi$, is the area in which vane c is moving to the inside. The space in between the rotor and the chamber is decreasing and therefore water is pressed in this area. The kidney shaped pressure opening is lying in this sector and covers the area from 1 ¼ π up to 1 ¾ π.

Because both the suction and the pressure opening cover an area of ½ π, in between these areas there are also two areas which cover ½ π. Therefore there will be always at least one vane in between the suction and the pressure openings.

A prerequisite of the shape of the chamber in the housing is that the strips have minimal clearance for every position of the rotor. If this is realised, there will be minimal internal leakage of water, even at low rotational speeds. The chamber can't have the shape of a cylinder because for a cylinder, the available space for the position of vanes b and d is smaller than for the position of vanes a and c. For the determination of the shape, the chamber is seen as an internal cam roller. The radius at the end of each vane is replaced by a roll with the same radius. The angle over which the roll is moving is taken in radials. The shape of the left and the right part of the chamber must be such that the hart of the roll moves with the same distance to the outside in the right sector as it moves to the inside in the left sector. This is the case if for both sectors the same mathematical function is used which is 180° rotary symmetrical around the middle of the function.

In principle four functions fulfil this need being: a cosine, an inclined sine, a fifth degree polynomial and a seventh degree polynomial. For all four functions the radial speed is zero for φ = 0 and for φ = π. The cosine has the maximum acceleration for the beginning and the end of the function. But as both functions for the right and the left sector are symmetrical around the line through φ = 0 and φ = π, the same cosine function is needed for the left and for the right sector. Therefore there will be no jump in the acceleration at φ = 0 and φ = π. The cosine function has the lowest maximal acceleration and the smallest pressure angle in between the roll and the curve in the chamber. It also has the simplest mathematical description. The cosine function has therefore been chosen.

The diagram for the movement s, the speed v and the acceleration a as a function of φ is given in figure 2.

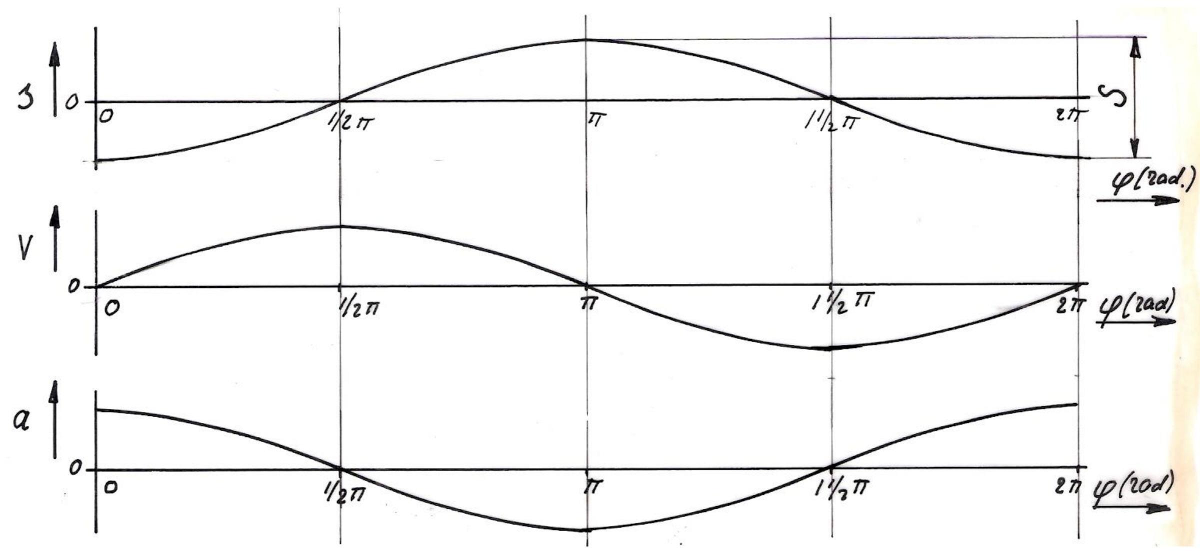

fig. 2 Variation s-φ, v-φ en a-φ curves for the centre of the roll which replaces the radius at the end of each vane.

Now the question is, what mathematical function has to be used for the curve of the left and the right section. It concerns the displacement function of the hart of the roll. The real shape of the curve which is followed by the hart of the roll is the displacement function of the hart of the roll unrolled on the so called basic circle.

The maximum stroke of a strip is chosen to be S. So the maximum stroke is the width of the chamber for $\varphi = \pi$. The displacement of vane a is called s and s = 0 for $\varphi = 0$. This is different from the s-φ curve given in figure 2. As the speed must vary according to a sine function and as the speed is the first derivative of the displacement, the displacement must be a $-\cos\varphi$ function. After some try and error it is found that the displacement s as a function of φ is given by:

$$s = f(\varphi) = ½ S – ½ S \cos\varphi = ½ S (1 – \cos\varphi) \quad (1)$$

This gives as the first and the second derivative that:

$$f'(\varphi) = ½ S \sin\varphi \quad (2)$$

$$f''(\varphi) = ½ S \cos\varphi \quad (3)$$

$f'(\varphi)$ is representative for the variation of the speed v and $f''(\varphi)$ is representative for the variation of the acceleration a. However, to find the absolute values of v and a one has to take the derivative to the time t in stead of the angle φ. But for these derivatives, the rotational speed has to be taken into account. This isn't useful to determine the shape of the curve of the chamber.

Formula 1 is now checked for the beginning, the middle and the end of the curves of the right and the left section. $\varphi = 0$ rad gives s = 0. This is right. $\varphi = ½ \pi$ rad gives s = ½ S. This is right because here we are half way the right curve. $\varphi = \pi$ rad gives s = S. This is right. $\varphi = 1 ½ \pi$ rad gives s = ½ S. This is right because here we are half way the left curve. $\varphi = 2\pi$ rad gives s = 0. This is right because here we are at the beginning again.

4 Determination of the stroke volume and the flow

The theoretical stroke volume of the pump $\nabla_{p\ th}$ is the volume of water pumped during one revolution when there is no internal leakage. It is very tricky to determine the stroke volume exactly. This is because the stroke of the vanes varies continuously and because a small part of the pumped water is returned from the pressure side to the suction side. To be able to find a reasonable approximation, the rotor is drawn such that vane a coincides with $\varphi = ¼ \pi$ (see figure 3). The suction and pressure openings are separated by two vanes for this position.

For a quart revolution, the volume of water which is contained in the space in between the rotor, the housing and the vanes b and c is pumped from the suction side to the pressure side. For a quart revolution, the volume of water which is contained in the space in between the rotor, the housing and the vanes d and a is pumped back from the pressure side to the suction side. So for a quart revolution the difference in between both volumes is pumped from the suction to the pressure side and four times this difference is pump during one complete revolution.

To be able to determine the volumes in between the vanes b and c and in between the vanes d and a, the areas in the plane of rotation in between the vanes, the rotor and the housing have to be determined. The problem for this determination is that s varies. To find a simple formula for $\nabla_{p\ th}$, an arc is drawn in the upper section with the hart of the rotor as centre and with such radius that the arc just touches the points where the vanes b and c are touching the chamber (see figure 3).

For the right point at $\varphi = \frac{3}{4}\pi$, using formula 1, it can be determined that $s = \frac{1}{2} S (1 - \cos 3/4\, \pi) = 0.8536\ S$. The distance in between the arc and the curve in the housing for $\varphi = \pi$ then is $S - 0.8536\ S = 0.1464\ S$.

The area in between the vanes b and c is now divided into a part with a constant width of 0.8536 S and a moon shaped part with a maximum width of 0.1464 S.

The area in between the vanes d and a is formed by two half moon shaped areas which also have a maximum width of 0.1464 S. The summed length of both parts is somewhat shorter than the length of the moon shaped part in between the vanes b and c because the radius is smaller and because the vane is at the widest part. Therefore the area is also somewhat smaller. But if this difference is neglected, the amount of water pumped from the pressure side to the suction side is proportional to the moon shaped area in between the vanes b and c. The net amount of water pumped from the suction side to the pressure side, then is proportional to the area in between the vanes b and c with the width 0.8536 S.

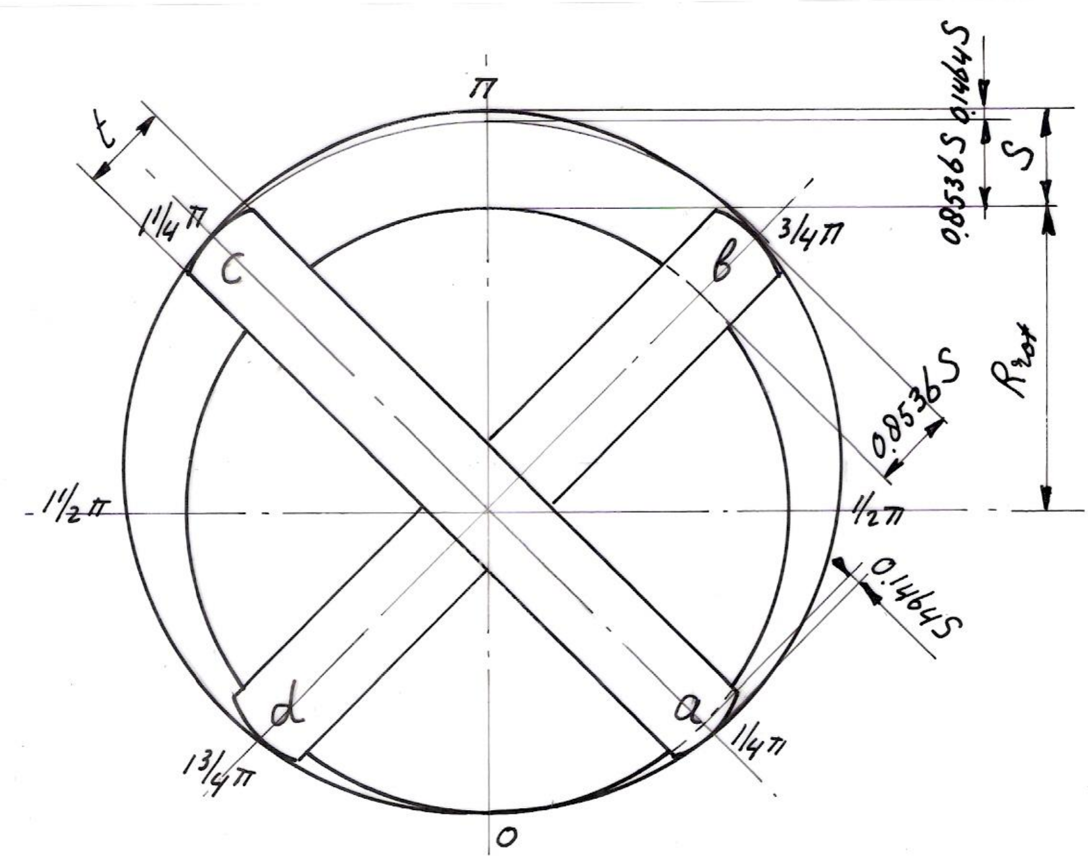

fig. 3 Rotor drawn for the position that vane a coincides with $\varphi = \frac{1}{4}\pi$

The thickness of a vane is chosen t. The width of a vane is identical to the width of the chamber in the housing and this width is chosen h. A vane has a tip radius R_{vane} which is identical to the radius of the roll for which the chamber is designed. R_{vane} must be smaller than the smallest radius of the chamber to prevent undercut. Therefore, R_{vane} is certainly smaller than the radius of the chamber at the positions $\varphi = \frac{1}{4}\pi$ and $\varphi = \frac{3}{4}\pi$. Therefore there will be two very small volumes at both sides of the contact line in between a vane and the chamber which also are a part of the pumped volume. These volume are neglected and so it is assumed that a vane makes contact with the chamber over the whole thickness of the vane. The area of a vane which is jutting out of the rotor at $\varphi = \frac{3}{4}\pi$ then is about $0.8536 * t * S$. In the following formulas all measures are taken in m.

For the theoretical stroke volume $\nabla_{p\ th}$ we now find that:

$$\nabla_{p\ th} = \{\pi\ [\ (R_{rot} + 0.8536\ S)^2 - R_{rot}^2] - 4 * 0.8536 * t * S\} * h \qquad \text{or}$$

$$\nabla_{p\ th} = [\pi\ (1.7072\ R_{rot} * S + 0.7286\ S^2) - 3.4144 * t * S] * h \qquad (m^3) \qquad (4)$$

Because of the volumetric efficiency η_{vol}, the real stroke volume ∇_p is smaller than $\nabla_{p\ th}$. The volumetric efficiency is determined by the gaps in between the rotor, the covers, and the vanes and by the head and the rotational speed. For certain gaps, the leaking flow is almost independent of the rotational speed. The volumetric efficiency will therefore become lower as the rotational speed drops. It is assumed that the pump can be designed such that the volumetric efficiency is about 0.95 for the design rotational speed and the design height. The real stroke volume ∇_p is given by:

$$\nabla_p = \nabla_{p\ th} * \eta_{vol} \qquad (m^3) \qquad (5)$$

The flow q is the real stroke volume times the number of revolutions per second of the pump. The number of revolutions of the pump n is normally given in rpm. This gives:

$$q = \nabla_p * n / 60 \qquad (m^3/s) \qquad (6)$$

(4) + (5) + (6) gives:

$$q = [\pi\ (1.7072\ R_{rot} * S + 0.7286\ S^2) - 3.4144 * t * S] * h * \eta_{vol} * n / 60 \qquad (m^3/s) \quad (7)$$

This is the end of the translation of report KD 109 from Dutch into English. The remaining part deals with the VIRYA-3B4 windmill to which the pump was coupled. For coupling of the pump to an electric motor this part is not relevant.

5 Description of the electric motor

A permanent magnet DC motor is preferred because of the high efficiency but such motor is rather expensive and may be difficult to obtain in developing countries. So in the first instance it is chosen to use a standard 370 W, 1-phase or 3-phase, 4-pole asynchronous motor frame size 71 which runs at a nominal rotational speed of about 1400 rpm. In this case, the direct current supplied by the solar panel or by the windmill has to be inverted to a 1-phase or 3-phase current or one has to use a windmill which is directly supplying 3-phase current. As the frequency of this current must be rather high, one has to use a multi pole generator if the generator is direct drive. A 22-pole generator of this kind is described in report KD 506 (ref. 3). The advantage of an asynchronous motor is that the outside dimensions are standardised and that therefore it isn’t necessary to select a certain brand.

There are manufactures of permanent magnet DC generators which make use of the same frame size as used for asynchronous motors. One of these manufacturers is the Dutch company Creusen from Roermond (see www.creusen.nl). Permanent magnet motors of this brand are supplied for frame size 56, 71 and 90. A range of voltages and powers is available for frame size 71. Assume the nominal voltage is 24 V and the nominal power is 350 W at a rotational speed of 1500 rpm. This requires motor housing 71L-2GP. If this motor is used in combination with a solar panel or a windmill, the real voltage will be higher than 24 V. It is assumed that a voltage controller is used which limits the voltage up to 28 V. This means that the maximum rotational speed is 1500 * 28 / 24 = 1750 rpm. This is a factor 1.25 higher than the rotational speed of a 4-pole asynchronous motor. The next calculations are given for the asynchronous motor with n = 1400 rpm.

Motors of frame size 71 are normally supplied with a foot B3 but can also be supplied by a flange. It is expected that they can also be supplied with a combination of foot and flange. This is easy because then the pump can be connected to the flange. At this moment it is chosen for a foot B3 with the motor axis horizontal and for a flange C105. 105 is the outside diameter of the flange in mm. It is provided by four threaded holes M6 at a pitch circle of 85 mm. It has a centralization collar with a diameter of 70 mm and a height of the collar of 2.5 mm.

A motor with frame size 71 has a shaft diameter d = 14 mm. The shaft diameter at the bearings is 15 mm so there is a 0.5 mm collar. The shaft length is 30 mm. There is a 5 mm key groove in the shaft. The key juts 2 mm out of the shaft.

The pump will be designed such that it has a hollow shaft with a key groove and that the motor axis can simply be pushed in the hollow shaft end of the pump. The pump must have a flange at the back side which is bolted to the flange of the motor by four M6 bolts.

6 Determination of the pump geometry and the output

It is assumed that the motor isn't used at the nominal power of 370 W but at a power of 320 W. So there is some reserve if the pump efficiency is lower than expected. The required electrical power depends on the motor efficiency and the cable losses. It is assumed that the motor efficiency is 0.65 and that the cable efficiency is 0.98. So the required electrical power is 320 / (0.65 * 0.98) = 502 W. The hydraulic power P_{hyd} is given by:

$$P_{hyd} = \rho_w * g * H * q \qquad \text{(W)} \qquad (8)$$

In this formula H is the total head. H is the sum of the static head H_{stat} and the dynamic head H_{dyn}. The static head is the height in between the water level in the well and the height of the outlet opening of the pressure pipe. The dynamic head is caused by the pipe losses in the suction and the pressure pipe. The dynamic head depends on the flow, the length of the pipes and very much on the inside pipe diameter. Calculation of the dynamic head is out of the scope of this report. It is assumed that the dynamic head is 10 % of the static head.

The required mechanical power at the motor shaft P depends on the pump efficiency η_p and is given by:

$$P = P_{hyd} / \eta_p \qquad \text{(W)} \qquad (9)$$

(8) + (9) gives:

$$P = \rho_w * g * H * q / \eta_p \qquad \text{(W)} \qquad (10)$$

(5) + (6) gives:

$$q = \nabla_{p\,th} * \eta_{vol} * n / 60 \qquad (\text{m}^3/\text{s}) \qquad (11)$$

(10) + (11) gives:

$$P = \rho_w * g * H * \nabla_{p\,th} * \eta_{vol} * n / (60 * \eta_p) \qquad \text{(W)} \qquad (12)$$

Formula 12 can be written as:

$$\nabla_{p\,th} = 60 * P * \eta_p / (\rho_w * g * H * \eta_{vol} * n) \qquad (\text{m}^3) \qquad (13)$$

It is assumed that the pump is designed for a static head H_{stat} = 40 m. It is assumed that the dynamic head H_{dyn} = 4 m. This gives a total head H = 44 m. Substitution of P = 320 W, η_p = 0.85, ρ_w = 1000 kg/m^3, g = 9.81 m/s^2, H = 44 m, η_{vol} = 0.95 and n = 1400 rpm in formula 13 gives that $\nabla_{p\ th}$ = 28.4 * 10^{-6} m^3 = 28.4 * 10^3 mm^3. This is a rather small stroke volume and the required pump will therefore be rather small.

Formula 4 gives the theoretical stroke volume. It can be seen that this stroke volume depends on four dimensions, the rotor radius R_{rot}, the maximum stroke S, the thickness of a vane t and the height h of the chamber. There are an infinite number of possibilities to realise a certain theoretical stroke volume. The vane grooves in the rotor are made with a finger cutter on a milling machine. To prevent that the cutter bends too much, the groove must be not very deep with respect to the width of the groove. It is assumed that the minimum value of t is ¼ of the groove depth h. The stroke volume increases almost linear with S. So the overall pump dimensions and the costs of the used materials decrease if S increases. However, the pressure angle and the acceleration forces increase at increasing S.

In report KD 109 a pump geometry has been chosen with R_{rot} = 40 mm, S = 13 mm, t = 12 mm and h = 40 mm. The theoretical stroke volume $\nabla_{p\ th}$ = 105.7 * 10^3 mm^3. So the required stroke volume is a factor 28.4 / 105.7 = 0.269 lower. If all dimensions of the pump are reduced by a factor 0.269 $^{1/3}$ = 0.646, the volume will be reduced by a factor 0.269. This results in R_{rot} = 25.8 mm, h = 25.8 mm, t = 7.8 mm and S = 8.4 mm. These calculated values aren’t nice.

Assume R_{rot} = 25 mm, h = 25 mm t = 8 mm and S = 9 mm. Substitution of these values in formula 4 gives $\nabla_{p\ th}$ = 28.5 * 10^3 mm^3 = 28.5 * 10^{-6} m^3. This is about the required theoretical stroke volume so the chosen values for R_{rot}, h, t and S are correct.

Substitution of $\nabla_{p\ th}$ = 28.5 * 10^{-6} m^3, η_{vol} = 0.95 and n = 1400 rpm in formula 11 gives q = 0.632 * 10^{-3} m^3/s = 2.27 m^3/hour = 54.6 m^3/day. This is a large amount of water and this calculation shows that the pump can be used for pumping drinking water and even for small scale irrigation.

The radius of the roll R_{roll} out of KD 109 was 15 mm. Multiplying of this value by the scale factor 0.646 gives R_{roll} = 9.7 mm. Assume R_{roll} = 10 mm.

In the calculations up to now it is assumed that there is no gap in between the rotor and the chamber at φ = 0. In reality this isn’t possible because the water of the left half moon shaped area in between vane d and vane a is pumped from the pressure side to the suction side. So there must be a little gap in between the rotor and the housing at φ = 0 otherwise an infinite pressure will be built up. So the real value of the rotor must be a little smaller than 25 mm. If the real value is chosen 24.7 mm, the rotor diameter will be 49.4 mm and this means that the rotor can be machined from 50 mm stainless steel bar. But for the further calculations, the value R_{rot} = 25 mm will be used.

To make the shape of the chamber one needs a programmable milling machine. One has to use a finger cutter with a radius R = R_{roll} = 10 mm, so with a diameter of 20 mm. This is a very large diameter for a chamber height of 25 mm, so milling of the chamber can be done very stable. The stroke s = 0 for φ = 0. The shape of the curve which has to be followed by the hart of the cutter is the displacement function of the roll unrolled on the basic circle. This means that the diameter of the basic circle is 30 mm for R_{roll} = 10 mm and for R_{rot} = 25 mm.

If a programmable milling machine isn’t available, it might be possible to make the chamber using a turntable. If the pump housing is connected to the turntable with an eccentricity of exactly 4.5 mm and if a 20 mm cutter is used with the correct position, the required chamber shape can be realised by 360° rotation of the turntable.

The groove in the rotor has a width t = 8 mm and a height h = 25 mm. So the ratio h / t = 3.125 and this ratio is small enough for making an accurate groove with parallel sides.

R_{rot} = 25 mm and S = 9 mm. This means that the length of strip is 2 * 25 + 9 = 59 mm. Each strip is provided by a cut-away in the middle to prevent that the strips make contact. The width of this cut away is about 18 mm and the depth is 12.5 mm.

So the strip volume is about 50 * 25 * 8 = 10000 mm^3. The strips are made out of a good quality plastic like PUM or polyoxmethyleen (supplied as Delrin, Ertacetal or Hostaform). The density ρ is about 1.4 * 10^3 kg/m^3 or 1.4 * 10^{-6} kg /mm^3. So the mass of a strip is about 0.014 kg = 14 gram which is very low. If PUM is wearing too fast one may use carbon imbedded Teflon.
The housing is made out of stainless steel bar with a diameter of 100 mm. A flange is made at the back side to be able to connect the pump to the motor flange. The rotor shaft must have a diameter of at least 25 mm if it is provided with a 14 mm hollow shaft and with a key groove for the motor shaft. So the geometry of the housing and the shaft differs from the geometry as given in figure 1.

The pump has no valves and small gaps in between the internal components. So the water will slowly flow down to the well once the motor is stopped. Another aspect is that the electric motor may turn in the opposite direction because the pump can work as a hydraulic motor driven by the pressure drop over the vanes. Both effects can be prevented if a foot valve is placed at the entrance of the suction line. But one must be alert that the motor never turns in the opposite direction if a foot valve is placed because this will blow the suction line or the pump or it will burn the motor winding!

A vane pump needs clean water with no sand particles. So a filter with a sufficient area to prevent a large pressure drop has to be placed at the entrance of the suction line.

7 Calculation of the maximum acceleration force F_{max} acting on a strip

The second derivative f''(φ) given by formula 3 is representative for the acceleration. However, to find the real acceleration a, one has to take the derivative to t in stead of to φ. The relation in between φ, ω and t is given by:

$$\varphi = \omega * t \qquad (s) \qquad (14)$$

The relation in between the angular velocity ω and the rotational speed n in rpm is given by:

$$\omega = \pi * n / 30 \qquad (rad/s) \qquad (15)$$

(1) + (14) gives:

$$s = f(t) = ½ S – ½ S \cos(\omega * t) = ½ S [1 – \cos(\omega * t)] \qquad (m) \qquad (16)$$

This gives as the first and the second derivative that:

$$v = f'(t) = ½ S * \omega \sin(\omega * t) \qquad (m/s) \qquad (17)$$

$$a = f''(t) = ½ S * \omega^2 \cos(\omega * t) \qquad (m/s^2) \qquad (18)$$

The acceleration a has an extreme value for φ = ω * t = 0 and for φ = ω * t = π (see figure 2). Both extremes have the same absolute value so any of the two values for φ can be taken. Substitution of ω * t = 0 in formula 18 gives for the maximum acceleration a_{max} that:

$$a_{max} = ½ S * \omega^2 \qquad (m/s) \qquad (19)$$

The maximum acceleration force F_{max} is given by:

$$F_{max} = a_{max} * m \qquad (N) \qquad (20)$$

(19) + (20) gives:

$$F_{max} = \tfrac{1}{2} S * \omega^2 * m \qquad (N) \qquad (21)$$

It was assumed that the nominal rotational speed of the electric motor n = 1400 rpm. Substitution of n = 1400 rpm in formula 15 gives that ω = 146.6 rad/s. Substitution of S = 9 mm = 0.009 m, ω = 146.6 rad/s and m = 0.014 kg in formula 21 gives F_{max} = 1.35 N. This is a very low force, so moving of the strip will cause only little friction in between the strip and the chamber.

8 Determination of the bearings, the seal and the hose pillars (see figure 4 page 15)

Most centrifugal pumps use a so called mechanical seal to prevent that the water is leaking out of the pump at the shaft. A mechanical seal is a rather complicated component for which the seal area is perpendicular to the rotor shaft. The mechanical seal is mounted in the pump housing as close as possible to the rotor and locked by a separate flange. Mostly ball bearings are used for the rotor shaft. The bearings are mounted in the dry part of the pump. For the vane pump, a different seal and different bearings are chosen.

It is chosen to use sealed needle bushes of manufacture INA with an inside diameter of 25 mm. The front bearing has a length of 30 mm. The bearing code is HK2530.2RS. The back bearing has a length of 20 mm. The bearing code is HK2520.2RS. The distance in between both bearings is 28 mm. The bearings are used without inner rings so they are running directly on the stainless steel shaft. It is assumed that normal stainless steel is hard enough for the given load and rotational speed.

In stead of a mechanical seal, a Garlock PS-seal is chosen. This seal has the same dimension as a normal oil seal. It has a sealing lip which runs on the rotor shaft. The shaft diameter at the seal is 25 mm. The seal code is Garlock PS-seal 25 * 35 * 8. The sealing lip is made of a material called GYLON which is a modified PTFE. The shaft should be very smooth and rather hard. It is expected that normal stainless steel is hard enough for the chosen pump head and the chosen rotational speed of the pump. Information about this seal can be found on: www.eriks.nl/documentatie/afdichtingen/asafdichtingen/garlock-ps-seal.pdf. Tests have to prove if standard stainless steel is hard enough for the seal and for the bearings. If not, the shaft has to be covered by a hard layer which may be chromium or ceramics.

The inside diameter of the lip is much smaller than the shaft diameter when the seal isn't mounted. Therefore the shaft must have a tapered end when the shaft is mounted such that the shaft enters the seal from the pressure side. It is advised to use a cone angle of 10° and to make the smallest diameter of the cone 5 mm smaller than the diameter of the shaft. A cone angle of 15° might also be allowed and this results in a tapered length of about 9 mm.

The rotor shaft is mounted directly to the motor shaft. The motor shaft has a diameter of 14 mm and a length of 30 mm and it is provided with a 5 mm key which juts 2 mm out of the shaft. This key requires an internal key groove in the motor shaft. To be able to make this key groove a chamber with a diameter of 19 mm is required at the end of the hole. The key groove requires a shaft diameter of at least 25 mm at the bearings and that's why this diameter is chosen. This shaft has a 4 mm wide, 45° tapered cone at the back side of the rotor to make the connection of the rotor and the shaft strong enough.

The pump head results in a pressure difference Δp over the rotor. The pressure difference results in a force F which pushes the rotor to one side. This force is taken mainly by the front bearing. The pressure difference Δp is given by:

$$\Delta p = \rho_w * g * H \qquad (N/m^2) \qquad (22)$$

Substitution of ρ_w = 1000 kg/m^3, g = 9.81 m/s^2 and H = 44 m in formula 22 gives Δp = 431640 N/m^2 = 0.43164 N/mm^2.

This pressure difference is working over the rotor which has a diameter of 50 mm and a thickness of 25 mm. So the area of the rotor A_r = 25 * 50 = 1250 mm^2.

The force F is given by:

$$F = \Delta p * A_r \qquad \text{(N)} \qquad (23)$$

Substitution of Δp = 0.43164 N/mm^2 and A_r = 1250 mm^2 in formula 23 gives F = 540 N. This force is working in the middle of the rotor, so at a distance of 12.5 mm from the bottom of the chamber in the housing.

The distance f_1 in between the middle of the rotor and the middle of the back bearing f_1 = 98.5 mm. The distance f_2 in between the middle of the front bearing and the middle of the back bearing f_2 = 53 mm. The reaction force working on the front bearing is called F_A. The reaction force working on the back bearing is called F_B. Balance of moments around the hart of the back bearing gives:

$$F_A = F * f_1 / f_2 \qquad \text{(N)} \qquad (24)$$

Substitution of F = 540 N, f_1 = 98.5 mm and f_2 = 53 mm in formula 25 gives F_A = 1004 N.
$F_B = F_A – F$, so F_B = 464 N. So the load on the front bearing is a factor 2.164 larger than the load on the back bearing. The width of the front bearing is chosen 30 mm and the width of the back bearing is chosen 20 mm. The dynamic load factor C and the static load factor C_0 are given in the INA catalogue.
For bearing HK2530.2RS it is given that C = 25500 N and C_0 = 45000 N.
For bearing HK2520.2RS it is given that C = 15600 N and C_0 = 24000 N.
The ratio in between C = 1.635. The ratio in between C_0 = 1.875. Both ratios are smaller than 2.164 which indicates that the front bearing is loaded heaviest. The calculated load on the front bearing F_A = 1004 N. This is very much lower than C_0 so the static load is absolutely no problem. The dynamic load factor C is used to calculate the lifetime of the bearing. The lifetime of a needle bearing in hours L_h is given by formula:

$$L_h = 16666 * (C / P)^p / n \qquad \text{(hours)} \qquad (25)$$

In this formula P is the load F_A. The coefficient p is 10/3 = 3.3333 for needle bearings. n is the rotational speed in rpm. Substitution of C = 25500 N, P = 1004 N, p = 3.3333 and n = 1400 rpm in formula 25 gives L_h = 573251 hours = 65.4 year. So the front bearing is certainly strong enough. The lifetime of the back bearing will be even longer. Concerning the required lifetime, lighter bearings could have been used but it is nice to have a large reserve because the bearings are used directly on a stainless steel shaft which isn’t hardened.

The Garlock PS-seal can have a nominal maximum pressure of 10 bar which corresponds to a head H = 100 m but the real maximum pressure depends on the circumference speed v of the shaft. This speed is given by:

$$v = d * \pi * n / (60 * 10^3) \qquad \text{(m/s)} \qquad (26)$$

Substitution of d = 25 mm and n = 1400 rpm in formula 26 gives v = 1.83 m/s. The allowable pressure depends on the type of GYLON used for the lip of the seal. Graphs for two types of GYLON are given being GYLON-B and GYLON-W.

GYLON-B is the standard material which is used for the chosen seal. In the graph it can be read that the allowable pressure for GYLON-B and v = 1.83 m/s is 10 bar. The maximum allowable speed for a pressure of 10 bar is even about 4 m/s so there is a large reserve. It is expected that a seal made out of GYLON-B will work if the roughness of the shaft is made low enough. After certain time of operation it has to be checked if the stainless steel is not worn at the position of the seal.

It has to be prevented that the front bearing becomes wet if some water is leaking along the seal. Therefore a hole is drilled in the bottom of the housing which ends in the space in between the seal and the front bearing. An O-ring is used in between the housing and the front cover.

The suction and pressure hoses are connected to hose pillars screwed in the cover. ½" hose pillars are drawn in figure 4. These hose pillars have an inner diameter of 15 mm so an inner area $A_h = \pi/4 * 15^2 = 177 \text{ mm}^2 = 0.177 * 10^{-3} \text{ m}^2$. It is assumed that hoses with the same inner diameter are used. The water speed in the hose v_h is given by:

$$v_h = q / A_h \quad \text{(m/s)} \tag{27}$$

Substitution of $q = 0.632 * 10^{-3} \text{ m}^3/\text{s}$ and $A_h = 0.177 * 10^{-3} \text{ m}^2$ in formula 27 gives that $v_h = 3.6$ m/s. This is a rather high speed and I doubt if this is allowed. May be it is better to use ¾" hose pillars which have an inner diameter of 20 mm. This results in a water speed of about 2 m/s which is still rather high. This also requires ¾" hoses. In this case the cross sectional area of the kidney shaped groove in the cover also has to be enlarged by making it deeper. This deepening and the larger hose pillar requires a thicker cover as drawn in figure 4.

9 References

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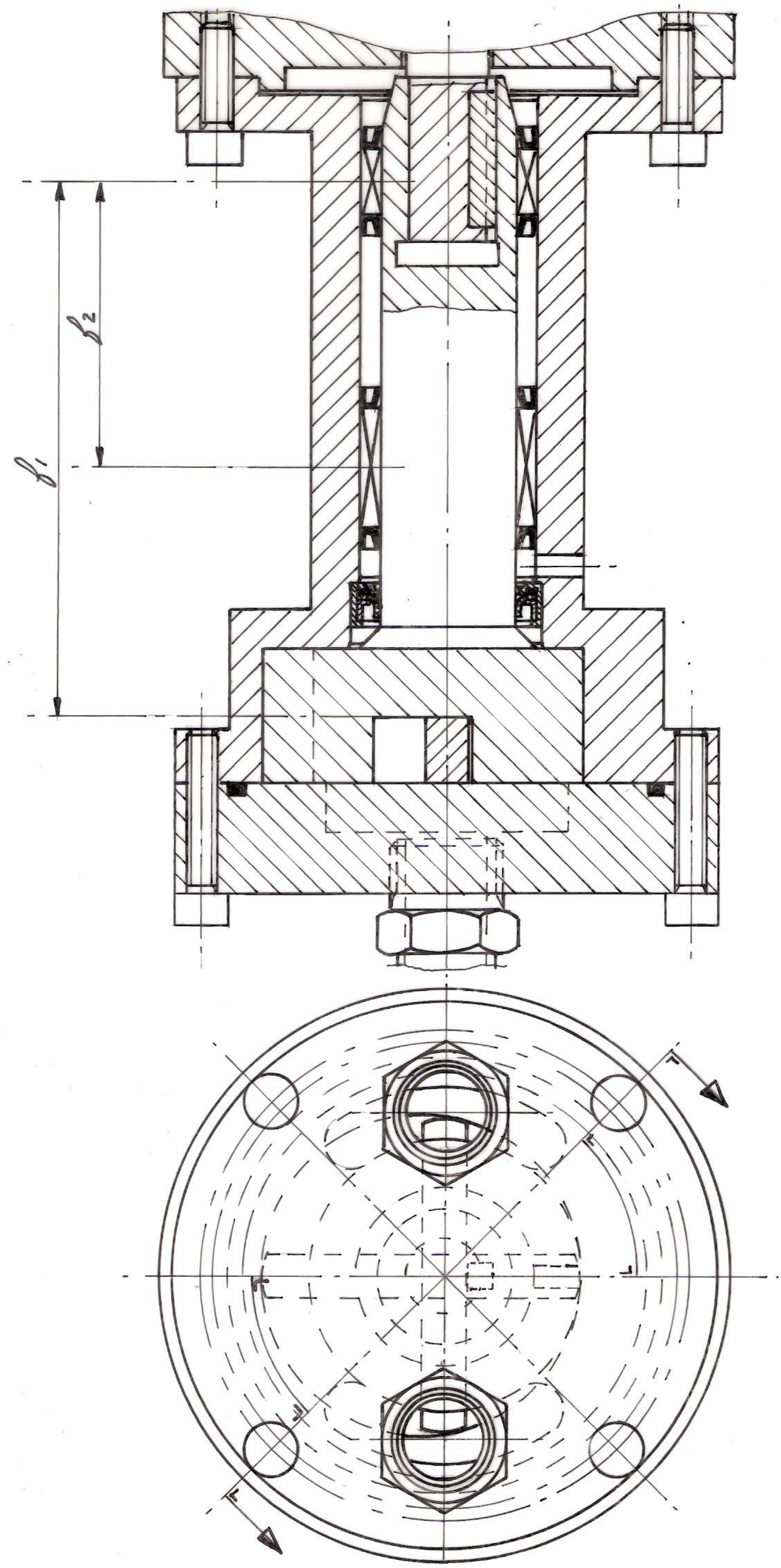

fig. 4 Vane pump for connection to a asynchronous 4-pole flanged motor size 71