Calculations executed for the 2-bladed rotor of the VIRYA-3B2 windmill (λ_d = 6.5, steel blades)

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KD 467

The rotor should not be used if the windmill has no safety system which turns the rotor fluently out of the wind at high wind speeds!

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1 Introduction

The VIRYA-3B2 windmill is developed for manufacture in developing countries. The VIRYA-3B2 has a 2-bladed rotor with galvanised steel blades and a design tip speed ratio λ_d = 6.5. The construction of the rotor is about similar to the construction of the VIRY-1.25 rotor which has been tested for three years and which has survived rotating, a measured wind speed of 26 m/s. However, the VIRYA-3B2 rotor is not made out of one strip like it is done for the VIRYA-1.25 rotor. The 2-bladed VIRYA-3B2 rotor is an alternative for the 2-bladed rotor with wooden blades of the VIRYA-3 which has a design tip speed ratio λ_d = 7. The VIRYA-3B2 is meant for 24 V or 12 V battery charging or for connection to a solar pump. The rotor drawing has not yet been made. For connection to the motor of a solar pump (without battery) it might be required to use a lighter vane blade to prevent too high voltages.

The generator is the same as the VIRYA-3 generator but recently an alternative generator using frame size 100 has been designed. The original 230/400 V winding is modified into a 115/200 V winding by connecting the first and second layer in parallel in stead of in series. The characteristics for 52 V star for the original 230/400 V winding will be the same as the characteristics for 26 V star for the modified 115/200 V winding. The characteristics for 26 V delta for the original 230/400 V winding will be the same as the characteristics for 13 V delta for the modified 115/200 V winding.

The VIRYA-3 generator is based on an asynchronous 2.5 kW, 3-phase motor of manufacture ROTOR type 5RN90L04V (with lengthened stator iron) for which the four pole short-circuit armature is replaced by an armature provided with permanent magnets. This generator has been measured extensively for the original winding (see report KD 78, ref. 1).

2 Description of the rotor of the VIRYA-3B2 windmill

The 2-bladed rotor of the VIRYA-3B2 windmill has a diameter D = 3 m and a design tip speed ratio λ_d = 6.5. Advantages of a 2-bladed rotor are that no welded spoke assembly is required and that the rotor can be balanced and transported easily.

The rotor has blades with a constant chord and no twist and is provided with a 7.14 % cambered airfoil. A blade is made of a galvanised steel strip with dimensions of 166 * 1250 * 2.5 mm and fifteen blades can be made from a standard sheet of 1.25 * 2.5 m with almost no material waste. Because the blade is cambered, the chord c is a little less than the blade width, resulting in c = 164 mm = 0.164 m. The blades are identical to the blades of the former 3-bladed VIRYA-3D windmill and can be cambered with the same blade press (given on drawing 0204-02).

The blades are connected to each other by a 1 m long central strip with a width of 80 mm and a thickness of 8 mm. As this strip is rather thin, the rotor is rather flexible and this flexibility neutralizes vibrations caused by the gyroscopic moment or by wind turbulence.

The overlap in between strip and blade is 0.25 m resulting in a rotor diameter of 3 m and in a free blade length of 1 m. This free blade length, in combination with a sheet thickness of 2.5 mm, is expected to be short enough to prevent flutter at high wind speeds. However, the connection in between the blade and the end of the central strip must be very tight. This is realised by making a radius to the backside of the strip identical to the radius of the blade camber and by an extra cambered sheet size 3 * 50 * 50 mm which is placed under the head of the outer connecting bolt. A sketch of the rotor is given in appendix 1.

The strip is twisted in between the hub and the blade root to realise the correct blade setting angle. Three M8 bolts are used for connection of blade and strip. These bolts can also be used for connection of the balancing weights.

The hub for the original VIRYA-3 generator is made of piece of round stainless steel bar with a diameter of 70 mm with a tapered hole in the middle for connection to the generator shaft. The hub for the alternative generator with frame size 100 and the original motor shaft is described in chapter 9.

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 2). This report (KD 467) has its own formula numbering. Substitution of $\lambda_d = 6.5$ and R = 1.5 m in formula (5.1) of KD 35 gives:

$\lambda_{r\,d} = 4.3333 * r$ (-) (1)

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$\beta = \phi - \alpha$ (°) (2)

$\phi = 2/3 \arctan 1 / \lambda_{r\,d}$ (°) (3)

Substitution of B = 2 and c = 0.164 m in formula (5.4) of KD 35 gives:

$C_l = 76.624\ r\ (1 - \cos\phi)$ (-) (4)

Substitution of V = 5 m/s and c = 0.164 m in formula (5.5) of KD 35 gives:

$R_{e\,r} = 0.547 * 10^5 * \sqrt{(\lambda_{r\,d}^2 + 4/9)}$ (-) (5)

The blade is calculated for six stations A till F which have a distance of 0.25 m of one to another. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l, α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The aerodynamic characteristics of a 7.14 % cambered airfoil are given in report KD 398 (ref. 3). The Reynolds values for the stations are calculated for a wind speed of 5 m/s because this is a reasonable wind speed for a windmill with V_{rated} = 11 m/s (see chapter 5). Those airfoil Reynolds numbers are used which are lying closest to the calculated values.

station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\,th}$ (-)	$C_{l\,lin}$ (-)	$R_{e\,r} * 10^{-5}$ V = 5 m/s	$R_e * 10^{-5}$ 7.14 %	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	$C_d/C_{l\,lin}$ (-)
A	1.5	6.5	5.8	0.164	0.59	0.52	3.57	3.4	-0.9	-1.2	6.7	7.0	0.069
B	1.25	5.417	7.0	0.164	0.71	0.77	2.99	3.4	-0.4	0	7.4	7.0	0.040
C	1	4.333	8.7	0.164	0.87	0.89	2.40	2.5	1.2	1.7	7.5	7.0	0.032
D	0.75	3.25	11.4	0.164	1.13	1.08	1.81	1.7	4.9	4.4	6.5	7.0	0.046
E	0.5	2.167	16.5	0.164	1.58	1.42	1.24	1.2	-	9.5	-	7.0	0.098
F	0.25	1.083	28.5	0.164	2.32	1.25	0.70	1.2	-	21.5	-	7.0	0.36

table 1 Calculation of the blade geometry of the VIRYA-3B2 rotor

No value for α_{th} and therefore for β_{th} is found for station E and F because the required C_l values can not be generated. The theoretical blade angle β_{th} varies only in between 7.5° and 6.5°. If the blade angle is taken 7° for the whole blade, the linearised blade angles are lying close to the theoretical values. The strip is twisted 7° to get the correct blade angle at the blade root.

4 Determination of the C_p-λ and the C_q-λ curves

The determination of the C_p-λ and C_q-λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.04. Figure 4.6 of KD 35 (for B = 2) en λ_{opt} = 6.5 and C_d/C_l = 0.04 gives $C_{p\ th}$ = 0.395. The blade is stalling at station F so only the part of the blade till a point 0.05 m outside station F is taken into account for the calculation of C_p. This gives k' = 1.2 m.
Substitution of $C_{p\ th}$ = 0.395, R = 1.5 m and blade length k = k' = 1.2 m in formula 6.3 of KD 35 gives $C_{p\ max}$ = 0.38. $C_{q\ opt} = C_{p\ max} / \lambda_{opt}$ = 0.38 / 6.5 = 0.0585.
Substitution of $\lambda_{opt} = \lambda_d$ = 6.5 in formula 6.4 of KD 35 gives λ_{unl} = 10.4.
The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - ½k) * C_l * c * k / \pi R^3 \quad (-) \qquad (6)$$

The blade angle is 7° for the whole blade. For a non rotating rotor, the angle of attack α is therefore 90° - 7° = 83°. The estimated C_l-α curve for large values of α is given as figure 5 of KD 398. For α = 83° it can be read that C_l = 0.23. During starting, the whole blade is stalling. So now the real blade length k = 1.25 m is taken.
Substitution of B = 2, R = 1.5 m, k = 1.25 m, C_l = 0.23 and c = 0.164 m in formula 6 gives that $C_{q\ start}$ = 0.0058. For the ratio in between the starting torque and the optimum torque we find that it is 0.0058 / 0.0585 = 0.1. This is acceptable for a rotor with λ_d = 6.5.

The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * ½\rho * \pi R^3} \right)} \quad (m/s) \qquad (7)$$

For the original VIRYA-3 generator with 5RN90L04V housing, a sticking torque Q_s of 0.4 Nm has been measured if the rotor is not rotating. Substitution of Q_s = 0.4 Nm, $C_{q\ start}$ = 0.0058, ρ = 1.2 kg/m^3 and R = 1.5 m in formula 7 gives that V_{start} = 3.3 m/s. This is acceptable for a 2-bladed rotor with a design tip speed ratio of 6.5 and a rated wind speed of 11 m/s. Because the generator is rectified in star for 24 V battery charging, the unloaded Q-n curve is rising only a little at increasing rotational speed. The C_q-λ curve of a rotor equipped with cambered steel blades is rising rather fast for V = 3.3 m/s. Therefore, the real starting wind speed will be about the same as the calculated value. This will be verified in chapter 8.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p-λ and C_q-λ curves can be determined if only two points of the C_p-λ curve and one point of the C_q-λ curve are known. The first part of the C_q-λ curve is determined according to KD 35 by drawing a S-shaped line which is horizontal for λ = 0.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 4). With this method, it can be determined that the C_q-λ curve is directly rising for low values of λ if a 7.14 % cambered sheet airfoil is used. This effect has been taken into account and the estimated C_p-λ and C_q-λ curves for the VIRYA-3B2 rotor are given in figure 1 and 2.

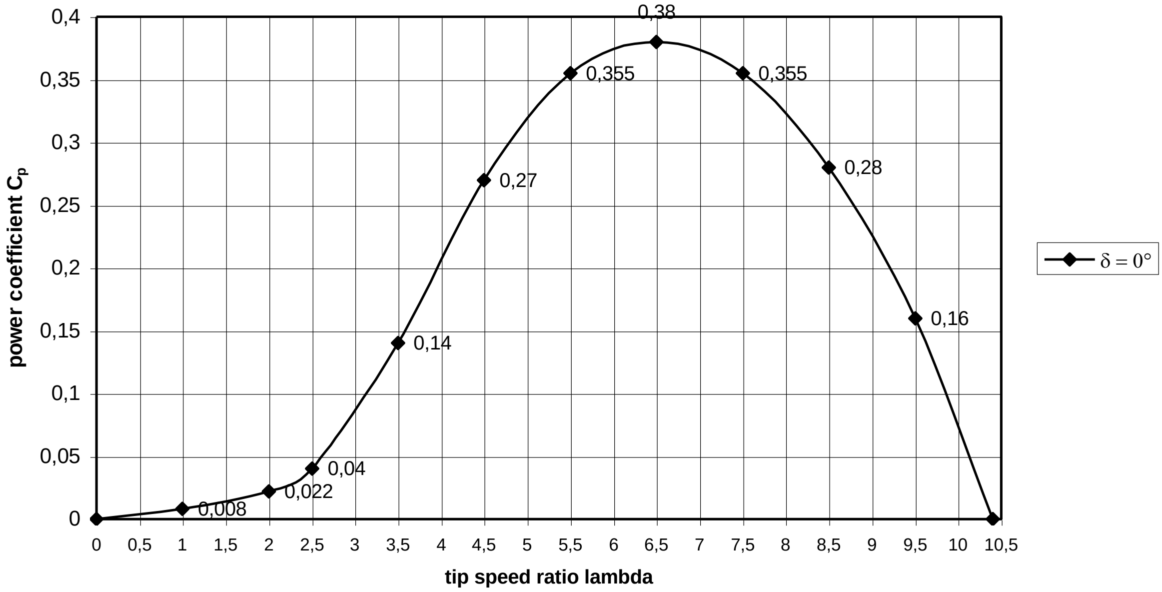

fig. 1 Estimated C_p-λ curve for the VIRYA-3B2 rotor for the wind direction perpendicular to the rotor ($\delta = 0°$)

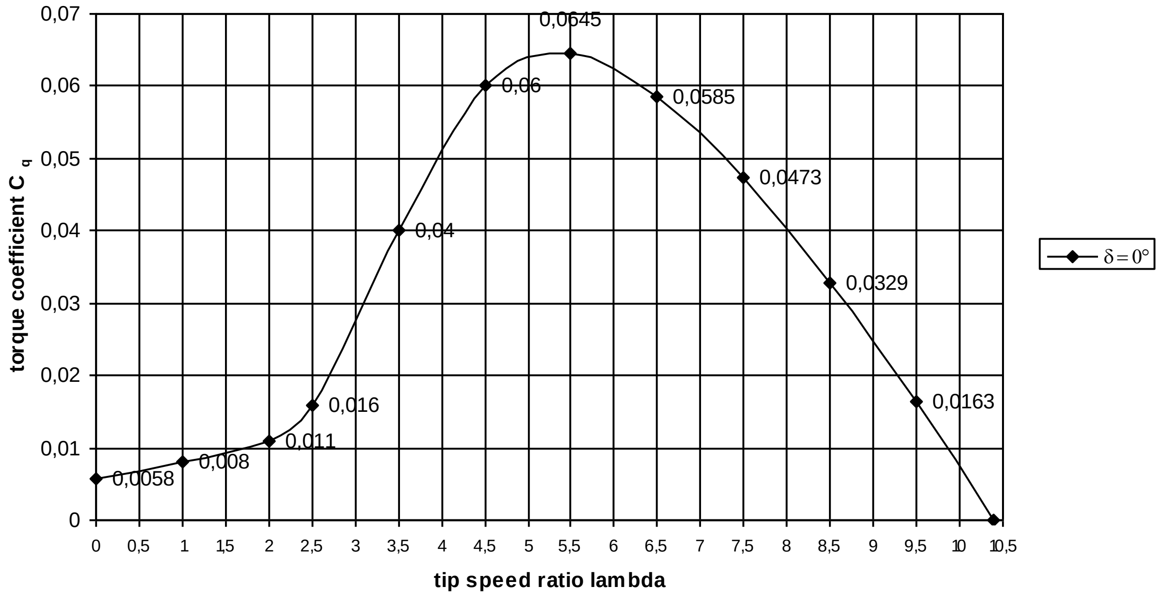

fig. 2 Estimated C_q-λ curve for the VIRYA-3B2 rotor for the wind direction perpendicular to the rotor ($\delta = 0°$)

5 Determination of the P-n curves and the P_{el}-V curves for 26 V star for a modified 115/200 V winding

The determination of the P-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_p-λ curve of the rotor and a δ-V curve of the safety system together with the formulas for the power P and the rotational speed n. The C_p-λ curve is given in figure 1. The δ-V curve of the safety system depends on the vane blade mass per area. The vane blade is made of 9 mm water proof plywood with a density of about 0.6 * 10^3 kg/m^3. In report KD 223 (ref. 5) a method is given to check the estimated δ-V curve and the estimated δ-V curve of the VIRYA-3.3D windmill is checked as an example. This windmill has also a vane blade made of 9 mm meranti plywood, so the δ-V curves of both windmills will be about the same. The estimated and calculated curves appear to lie very close to each other so it is allowed to use the estimated curve. The VIRYA-3.3D windmill has a rated wind speed of about 11 m/s. The estimated δ-V curve is given in figure 3.

The head starts to turn away at a wind speed of about 5 m/s. For wind speeds above 11 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The P-n curve for 11 m/s will therefore also be valid for wind speeds higher than 11 m/s.

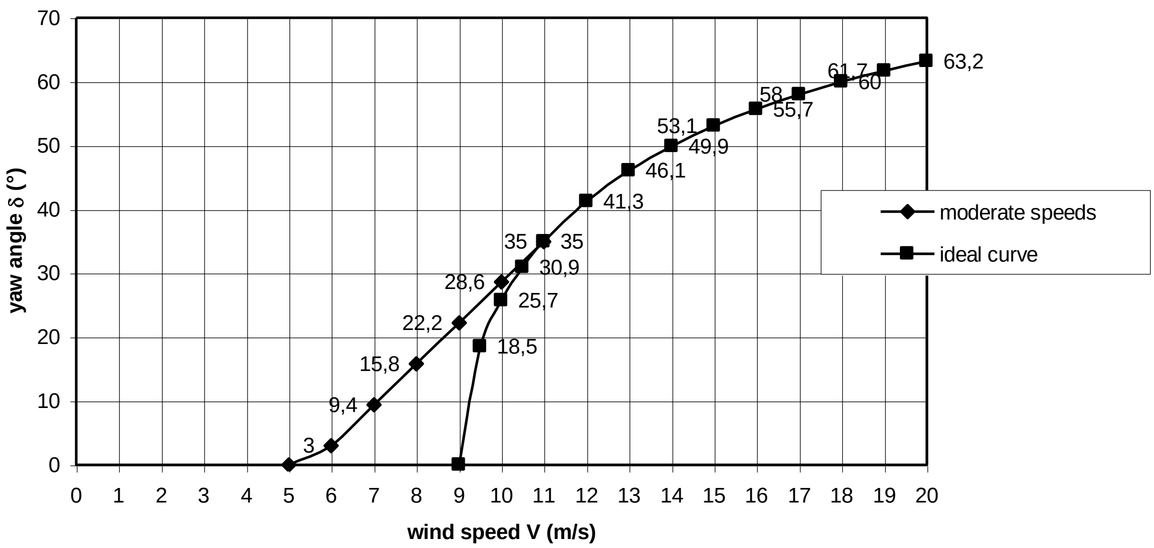

fig. 3 δ-V curve VIRYA-3B2 safety system with V_{rated} = 11 m/s

The P-n curves are used to check the matching with the P_{mech}-n curve of the generator for a certain gear ratio i (the VIRYA-3B2 has no gearing so i = 1). Because we are especially interested in the domain around the optimal cubic line and because the P-n curve for low values of λ appears to lie very close to each other, the P-n curves are not determined for low values of λ. The P-n curves are determined for wind the speeds 3, 4, 5, 6, 7, 8, 9, 10 and 11 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for P and n are used which are given in chapter 7 of KD 35.

Substitution of R = 1.5 m in formula 7.1 of KD 35 gives:

$n_\delta = 6.3662 * \lambda * \cos\delta * V$ (rpm) (8)

Substitution of ρ = 1.2 kg / m^3 en R = 1.5 m in formula 7.10 of KD 35 gives:

$P_\delta = 4.2412 * C_p * \cos^3\delta * V^3$ (W) (9)

The P-n curves are determined for C_p values belonging to λ = 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5 and 10.4. (see figure 1). For a certain wind speed, for instance V = 3 m/s, related values of C_p and λ are substituted in formula 8 and 9 and this gives the P-n curve for that wind speed. For the higher wind speeds the yaw angle as given by figure 5, is taken into account. The result of the calculations is given in table 2.

		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 3°		V = 7 m/s δ = 9.4°		V = 8 m/s δ = 15.8°		V = 9 m/s δ = 22.2°		V = 10 m/s δ = 28.6°		V = 11 m/s δ = 35°	
λ (-)	C_p (-)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)
3.5	0.14	66.8	16.0	89.1	38.0	111.4	74.2	133.5	127.7	153.9	195.6	171.5	270.8	185.7	343.6	195.6	402	200.8	434
4.5	0.27	85.9	30.9	114.6	73.3	143.2	143.1	171.7	246.3	197.8	377.2	220.5	522.3	238.7	662.6	251.5	775	258.1	838
5.5	0.355	105.0	40.7	140.1	96.4	175.1	188.2	209.8	323.9	241.8	495.9	269.5	686.8	291.8	871.2	307.4	1019	315.5	1102
6.5	0.38	124.1	43.5	165.5	103.1	206.9	201.5	247.9	346.7	285.8	530.8	318.5	735.1	344.8	932.5	363.3	1091	372.9	1179
7.5	0.355	143.2	40.7	191.0	96.4	238.7	188.2	286.1	323.9	329.7	495.9	367.5	686.8	397.9	871.2	419.2	1019	430.2	1102
8.5	0.28	162.3	32.1	216.5	76.0	270.6	148.4	324.2	255.5	373.7	391.1	416.5	541.7	450.9	687.1	475.1	804	487.6	869
9.5	0.16	181.4	18.3	241.9	43.4	302.4	84.8	362.4	146.0	417.7	223.5	465.6	309.5	504.0	392.6	531.0	459	545.0	496
10.4	0	198.6	0	264.8	0	331.0	0	396.7	0	457.2	0	509.7	0	551.7	0	581.3	0	596.6	0

table 2 Calculated values of n and P as a function of λ and V for the VIRYA-3B2 rotor

The calculated values for n and P are plotted in figure 4. The optimum cubic line which is going through the points of maximum power, is also drawn in figure 4.

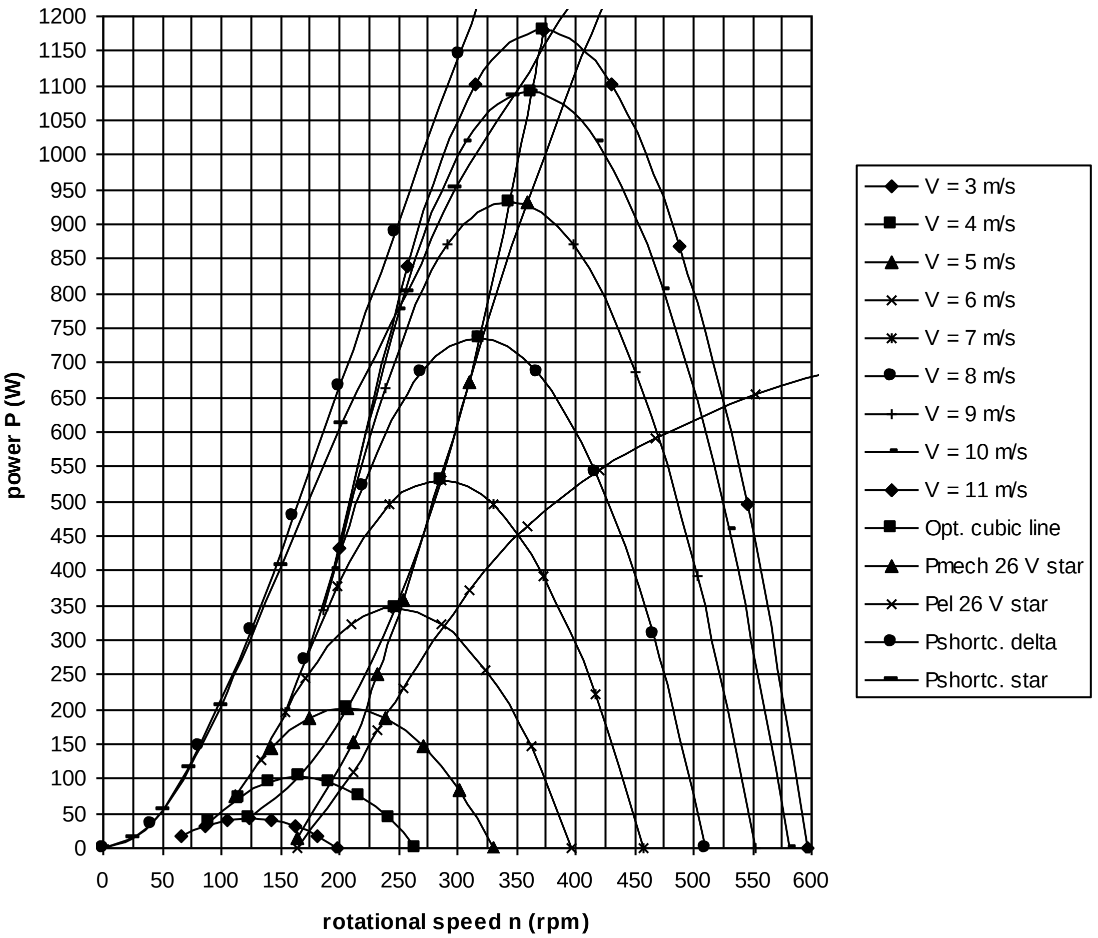

fig. 4 P-n curves of the VIRYA-3B2 rotor for V_{rated} = 11 m/s, optimum cubic line, P_{mech}-n and P_{el}-n curves of the generator for 26 V star and P-n curve for short-circuit in delta and star for the modified 115/200 V winding

The generator with 5RN90M04V housing has been measured with the original 230/400 V winding for 52 V, for rectification in star and the measurements are given in figure 2 and 3 of KD 78 (ref. 1). The original winding can be modified into a 115/200 V winding by connecting the first and the second layer in parallel in stead of in series. This procedure is described in report KD 341 (ref. 6). By this modification, the voltage halves and the current doubles. The characteristics for 26 V star for the modified 115/200 V winding are identical to the characteristics for 52 V star for the original 230/400 V winding. The P_{mech}-n and P_{el}-n curves for 26 V for the modified 115/200 V winding are copied in figure 4. A voltage of 26 V is the average charging voltage for a 24 V battery. The P-n curve for short-circuit in delta has also been measured and is also copied in figure 4. The P-n curve for short-circuit in star has also been measured and is also copied in figure 4.

The point of intersection of the P_{mech}-n curve for 26 V star of the generator with the P-n curve of the rotor for a certain wind speed, gives the working point for that wind speed. The electrical power P_{el} for that wind speed is found by going down vertically from the working point up to the point of intersection with the P_{el}-n curve. The values of P_{el} found this way for all wind speeds, are plotted in the P_{el}-V curve (see figure 5). For high powers, the voltage is higher than 26 V and therefore the generator efficiency will be higher too. This results in a somewhat higher electrical power. The P_{el}-V curve is corrected for this effect.

The matching of rotor and generator is good for wind speeds in between 5 and 11 m/s because the P_{mech}-n curve of the generator is lying close to the optimum cubic line. The P-n curve for short-circuit in delta is lying left from the P-n curve of the rotor for V = 11 m/s and higher. This means that the rotor will slow down to almost stand still for every wind speed if short-circuit in delta is made. The P-n curve for short-circuit in star is about touching the P-n curve of the rotor for V = 9.5 m/s (not drawn). So making short-circuit in star is not advised because the rotor can’t be stopped at high wind speeds and this finally will result in a burned generator winding! Short-circuit is star is the same as short-circuit in delta if the star point is short-circuited too. However, this requires an extra wire from the star point of the generator to the short-circuit switch and a short-circuit switch with three contacts.

The supply of power starts already at a wind speed of 3 m/s ($V_{cut\ in}$ = 3 m/s). This is rather low and therefore the windmill can be used in regions with low wind speeds. In chapter 4 it was calculated that V_{start} = 3.3 m/s so there is hysteresis in the P_{el}-V curve for wind speeds in between 3 and 3.3 m/s. The maximum power is 550 W which is rather good for a 3 m diameter rotor.

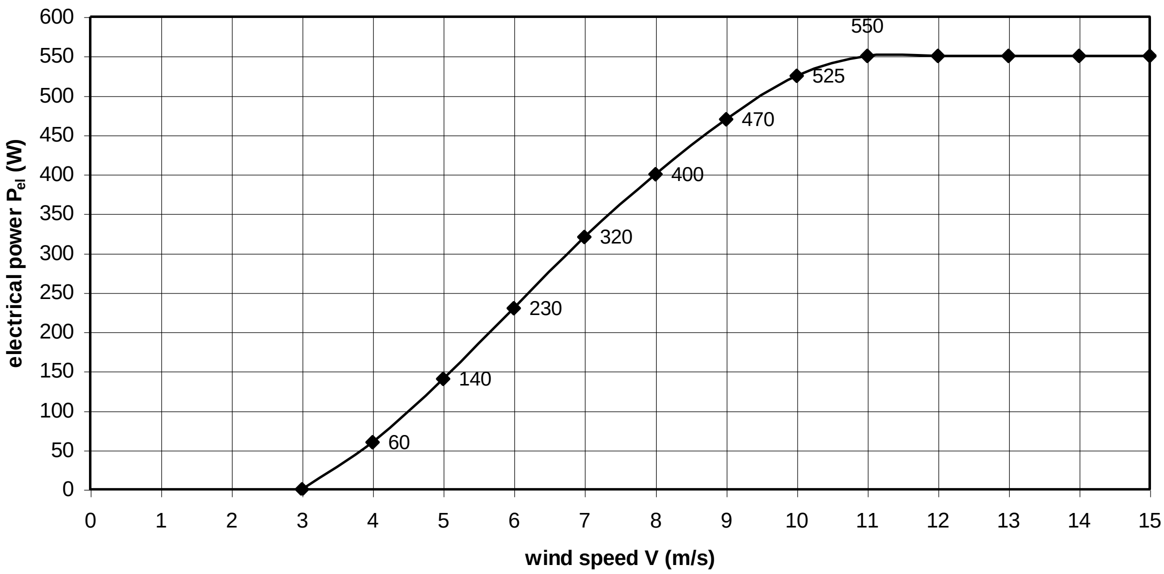

fig. 5 P_{el}-V curve of the VIRYA-3B2 windmill with V_{rated} = 11 m/s for 24 V battery charging and rectification in star for the modified 115/200 V winding

6 Determination of the P-n curves and the P_{el}-V curves for 13 V delta for a modified 115/200 V winding

It might be possible to use the VIRYA-3B2 for 12 V battery charging if the generator with a modified 115/200 V winding is rectified in delta. In the same way as it is done in chapter 5, the characteristics for 13 V delta are derived from the measurements for 26 V delta for the generator with 5RN90M04V housing. Figure 4 is copied as figure 6 and the characteristics for 26 V star are replaced by the characteristics for 13 V delta.

In figure 6 it can be seen that the P_{mech}-n curve for 13 V delta is lying somewhat left from the optimum cubic line. But the matching is good for wind speeds in between 5 and 11 m/s. The P_{el}-V curve for 13 V delta is made in the same way as it was done for 26 V delta and is given in figure 7.

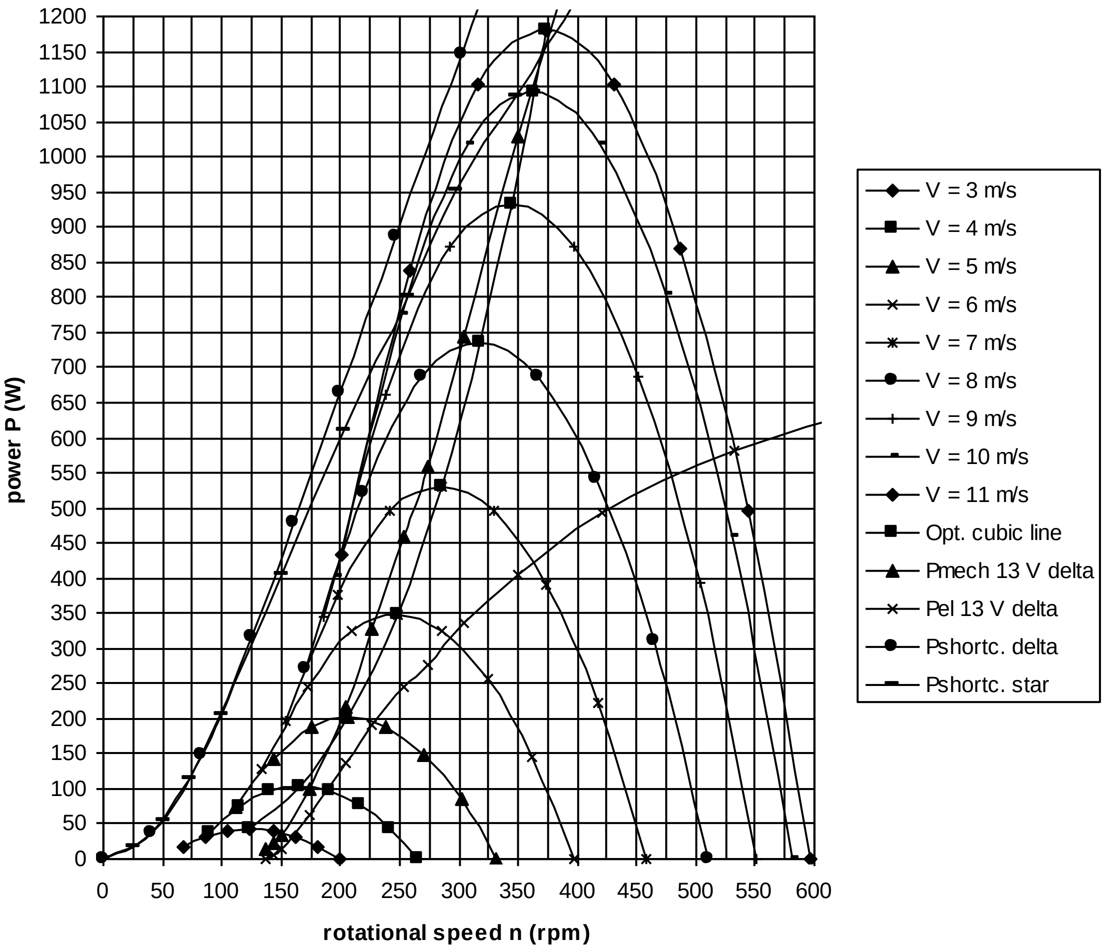

fig. 6 P-n curves of the VIRYA-3B2 rotor for V_{rated} = 11 m/s, P_{mech}-n and P_{el}-n curves of the generator for 13 V delta and P-n curve for short-circuit in delta and star for 5RN90M04 housing with a modified 115/200 V winding

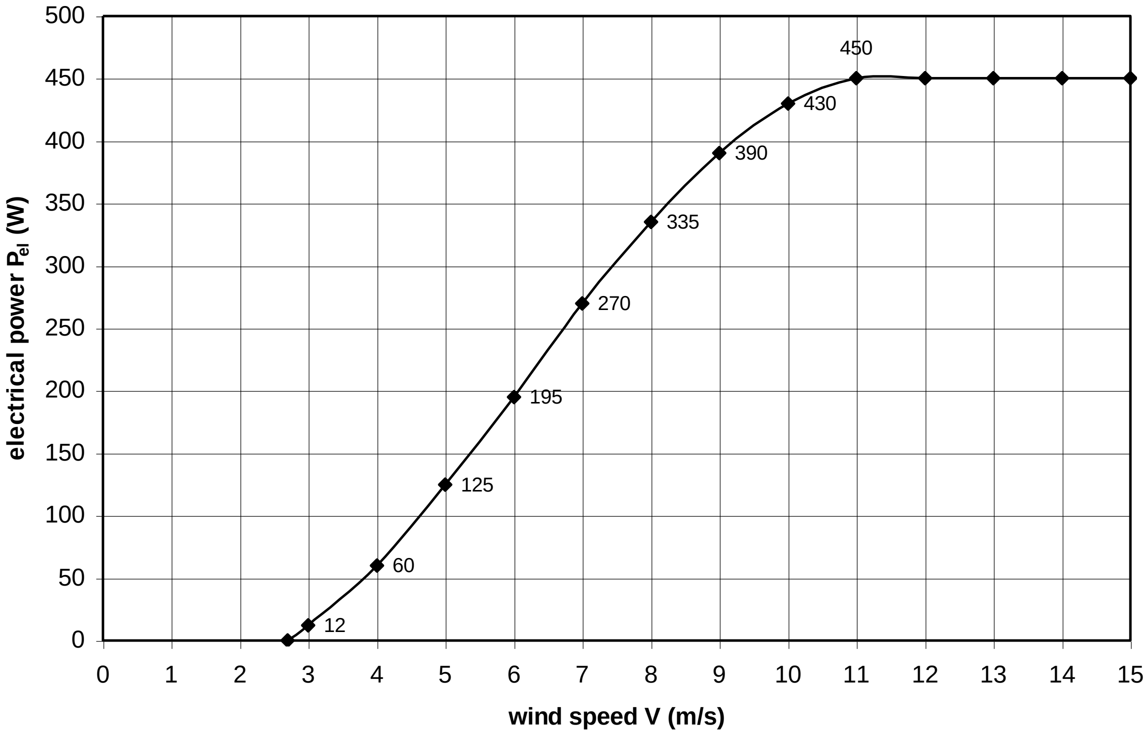

fig. 7 P_{el}-V curve of the VIRYA-3B2 windmill with V_{rated} = 11 m/s for 12 V battery charging and rectification in delta for a 5RN90M04 housing with a modified 115/200 V winding

The supply of power starts already at a wind speed of 2.7 m/s ($V_{cut\ in}$ = 2.7 m/s). This is low and therefore the windmill can be used in regions with low wind speeds. In chapter 4 it was calculated that V_{start} = 3.3 m/s so there is some hysteresis in the P_{el}-V curve for wind speeds in between 2.7 and 3.3 m/s. The maximum power is 450 W which is 100 W lower than for 26 V star. The P_{el}-V curve for 13 V delta is lying lower than the P_{el}-V curve for 26 V star for wind speeds higher than about 4 m/s which is caused by the lower generator efficiency in delta.

The original 230/400 V winding can be used for 24 V battery charging if the generator is rectified in delta. In this case one will find the same P_{el}-V curve as given in figure 7. But as the P_{el}-V curve for 26 V star for the modified 115/200 V winding is lying higher, use of the original 230/400 V winding and rectification in delta is not advised for 24 V battery charging. Another disadvantage of rectification in delta is that the starting behaviour will be lesser. This will be investigated in chapter 7. The original 230/400 V winding can be used for 48 V battery charging if the generator is rectified in star. In this case one will find the same P_{el}-V curve as given in figure 5. The cable losses for 48 V battery charging are rather small but 48 V requires a special voltage controller with two separated 27.6 V dump loads.

7 Checking of the starting behaviour

In chapter 4 it has been calculated that the starting wind speed is 3.3 m/s. At this wind speed the rotor is able to generate the unloaded sticking torque of 0.4 Nm of the generator if the shaft is not rotating. The unloaded sticking torque increases at increasing rotational speed but how much depends on how the generator is rectified. The sticking torque for rectification in delta rises faster than for rectification in star because higher harmonic currents can circulate in delta. The sticking torque has been measured for both situations for the 5RN90M04V housing and the result is given in figure 1 of report KD 78. The values for low rotational speeds are given in figure 8. Modification of the winding has no influence on the unloaded curves.

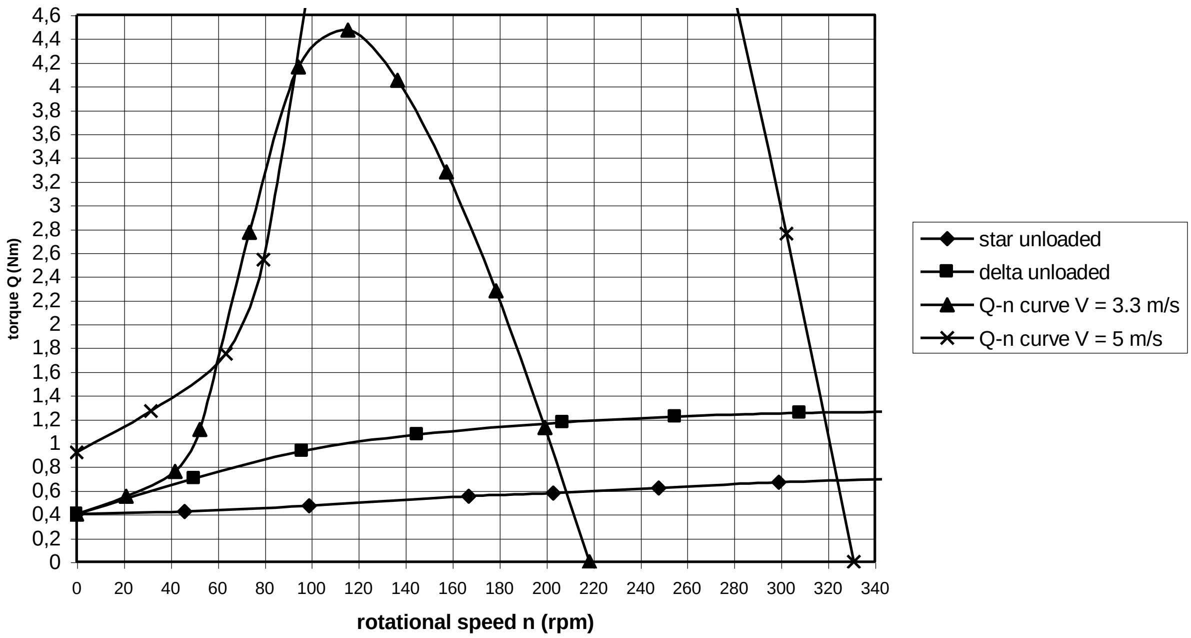

fig. 8 Unloaded sticking torque of the generator with modified for rectification in delta and star and Q-n curves of the rotor for V = 3.3 m/s and V = 5 m/s

The Q-n curve of the rotor for a certain wind speed can be determined in the same way as the P-n curve for a certain wind speed but one has to use the formula for Q in stead as the formula for P. The formula for Q for a rotor perpendicular to the wind is given by formula 4.3 of KD 35. Substitution of $\rho = 1.2$ kg/m^3 , V = 3.3 m/s and R = 1.5 m in this formula gives:

$$Q = 69.279 * C_q \qquad \text{(Nm)} \qquad (10)$$

The rotational speed n is given by formula 8. Substitution of $\delta = 0°$ and V = 3.3 m/s in formula 8 gives:

$$n = 21.008 * \lambda \qquad \text{(rpm)} \qquad (11)$$

The Q-n curve is now determined for values λ of 0, 1, 2, 2.5, 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5 and 10.4 and the corresponding values of C_q (see figure 2). The Q-n curve found this way is also given in figure 8. In figure 8 it can be seen that the Q-n curve for V = 3.3 m/s at low rotational speeds is rising faster than the unloaded sticking torque for rectification in star. The difference in between the Q-n curve of the rotor and the unloaded Q-n curve of the generator is used to accelerate the rotor. This means that the rotor will start at V = 3.3 m/s for rectification in star.

The Q-n curve for V = 3.3 m/s is also rising faster than the unloaded sticking torque for rectification in delta but the distance in between both curves is much smaller, especially for n < 40 rpm. So only a little torque difference is available to accelerate the rotor. This means that the real starting wind speed will be somewhat higher than 3.3 m/s and will be about 3.5 m/s which is still acceptable. So the starting behaviour in delta is acceptable but the starting behaviour in star is better.

If the wind speed is higher than 3.3 m/s, e.g. 5 m/s, a much higher torque difference is available to accelerate the rotor and for this wind speed, there will be almost no difference in the starting behaviour for star or for delta rectification. To illustrate this, the Q-n curve for V = 5 m/s is also given in figure 8.

8 Calculation of the strength of the blade strip

The blades are connected to each other by the central strip. This strip has a length l = 1000 mm, a width b = 80 mm and a height h = 8 mm. The blade has a thickness of 2.5 mm but the blade is cambered and the moment of resistance therefore increases substantially (see formula 12 report KD 398, ref. 3). The bending moment in the strip at the hub is also much higher than in the blade at the end of the strip. The strip is therefore the weakest component. The strip is loaded by a bending moment with axial direction which is caused by the rotor thrust and by the gyroscopic moment. The strip is also loaded by a centrifugal force and by a bending moment with tangential direction caused by the torque and by the weight of the blade but the stresses which are caused by these loads can be neglected.

Because the strip is long and rather thin, it makes the blade connection elastic and therefore the blade will bend backwards already at a low load. As a result of this bending, a moment with direction forwards is created by a component of the centrifugal force in the blade. The bending is substantially decreased by this moment and this has a favourable influence on the bending stress.

It is started with the determination of the bending stress which is caused by the rotor thrust. There are two critical situations:

1^e The load which appears for a rotating rotor at V_{rated} = 11 m/s. For this situation the bending stress is decreased by the centrifugal moment. The yaw angle is 35° for V_{rated} = 11 m/s.

2^e The load which appears for a slowed down rotor. The rotor is slowed down by making short-circuit in the generator winding. A graph has been made in which the Q-n curve of the rotor for V = 11 m/s has been plotted together with the Q-n curve of the generator for short-circuit in delta. For the working point it is found that the rotor rotates at a rotational speed of about 35 rpm and has a tip speed ratio of about 0.4. For this very low rotational speed the effect of compensation by the centrifugal moment is negligible and a tip speed ratio of 0.4 is very low. Therefore it is assumed that the rotor stands still.

8.1 Bending stress in the strip for a rotating rotor and V = 11 m/s

The rotor thrust is given by formula 7.4 of KD 35. The rotor thrust is the axial load of all blades together and exerts in the hart of the rotor. The thrust per blade $F_{t\,\delta\,bl}$ is the rotor thrust $F_{t\,\delta}$ divided by the number of blades B. This gives:

$$F_{t\,\delta\,bl} = C_t * \cos^2\delta * ½\rho V^2 * \pi R^2 / B \qquad \text{(N)} \qquad (12)$$

For the rotor theory it is assumed that every small area dA which is swept by the rotor, supplies the same amount of energy and that the generated energy is maximised. For this situation, the wind speed in the rotor plane has to be slowed down till 2/3 of the undisturbed wind speed V. This results in a pressure drop over the rotor plane which is the same for every value of r. It can be proven that this results in a triangular axial load which forms the thrust and in a constant radial load which supplies the torque. The theoretical thrust coefficient C_t for the whole rotor is 8/9 = 0.889 for the optimal tip speed ratio. In practice C_t is lower because of the tip losses and because the blade is not effective up to the rotor centre. The effective blade length k' of the VIRYA-3B2 rotor is only 1.2 m but the rotor radius R = 1.5 m. Therefore there is a disk in the centre with an area of about 0.04 of the rotor area on which almost no thrust is working. This results in a theoretical thrust coefficient C_t = 8/9 * 0.96 = 0.85. Because of the tip losses, the real C_t value is substantially lower. Assume this results in a real practical value of C_t = 0.75. It is assumed that the thrust coefficient is constant for values of λ in between λ_d and $\lambda_{unloaded}$.

Substitution of C_t = 0.75, δ = 35°, ρ = 1.2 kg/m^3, V = 11 m/s, R = 1.5 m and B = 2 in formula 12 gives $F_{t\,\delta\,bl}$ = 129 N.

For a pure triangular load, the same moment is exerted in the hart of the rotor as for a point load which exerts in the centre of gravity of the triangle. The centre of gravity is lying at 2/3 R = 1 m. Because the effective blade length is only k', there is no triangular load working on the blade but a load with the shape of a trapezium as the triangular load over the part R - k' falls off. The centre of gravity of the trapezium has been determined graphically and is lying at about r_1 = 1.05 m.

The maximum bending stress is not caused at the hart of the rotor but at the edge of the hub because the strip bends backwards from this edge. This edge is lying at r_2 = 0.024 m for the hub described in chapter 9. At this edge we find a bending moment $M_{b\ t}$ caused by the thrust which is given by:

$$M_{b\ t} = F_{t\ \delta\ bl} * (r_1 - r_2) \qquad \text{(Nm)} \qquad (13)$$

Substitution of $F_{t\ \delta\ bl}$ = 129 N, r_1 = 1.05 m and r_2 = 0.024 m gives $M_{b\ t}$ = 132.4 Nm = 132400 Nmm.

For the stress we use the unit N/mm^2 so the bending moment has to be given in Nmm. The bending stress σ_b is given by:

$$\sigma_b = M / W \qquad (\text{N/mm}^2) \qquad (14)$$

The moment of resistance W of a strip is given by:

$$W = 1/6\ bh^2 \qquad (\text{mm}^3) \qquad (15)$$

(14) + (15) gives:

$$\sigma_b = 6\ M / bh^2 \qquad (\text{N/mm}^2) \qquad (\text{M in Nmm}) \qquad (16)$$

Substitution of M = 132400 Nmm, b = 80 mm and h = 8 mm in formula 16 gives σ_b = 155 N/mm^2. For this stress, the effect of the stress reduction by bending forwards of the blade caused by the centrifugal force in the blade has not yet been taken into account. The gyroscopic moment has also not yet been taken into account.

Next it is investigated how far the blade bends backwards as a result of the thrust load and what influence this bending has on the centrifugal moment. Hereby it is assumed that the strip is bending in between the hub and the inner connection bolt of blade and strip. This bolt is lying at r_3 = 0.275 m = 275 mm. So the length of the strip l which is loaded by bending is given by:

$$l = r_3 - r_2 \qquad \text{(mm)} \qquad (17)$$

The load from the blade on the strip at r_3 can be replaced by a moment M and a point load F. F is equal to $F_{t\ \delta\ bl}$. M is given by:

$$M = F * (r_1 - r_3) \qquad \text{(Nmm)} \qquad (18)$$

The bending angle ϕ (in radians) at r_3 for a strip with a length l is given by (combination of the standard formulas for a moment plus a point load):

$$\phi = l * (M + ½\ Fl) / EI \qquad \text{(rad)} \qquad (19)$$

The bending moment of inertia I of a strip is given by:

$I = 1/12\ bh^3$ (mm^4) (20)

(17) + (18) + (19) + (20) gives:

$\phi = 12 * F * (r_3 - r_2) * \{(r_1 - r_3) + ½ (r_3 - r_2)\} / (E * bh^3)$ (rad) (21)

Substitution of $F = 129$ N, $r_3 = 275$ mm, $r_2 = 24$ mm, $r_1 = 1050$ mm, $E = 2.1 * 10^5$ N/mm^2, b = 80 mm and h = 8 mm in formula 21 gives: $\phi = 0.04068$ rad = 2.33°. This is an angle which can not be neglected. In report R409D (ref. 7) a formula is derived for the angle ε with which the blade moves backwards if it is connected to the hub by a hinge. This formula is valid if both the axial load and the centrifugal load are triangular. For the VIRYA-3B2 this is not exactly the case but the formula gives a good approximation. The formula is given by:

$$\varepsilon = \arcsin\left(\frac{C_t * \rho * R^2 * \pi}{B * A_{pr} * \rho_{pr} * \lambda^2}\right) \quad (°) \qquad (22)$$

In this formula A_{pr} is the cross sectional area of the airfoil (in m^2) and ρ_{pr} is the density of the used airfoil material (in kg/m^3). For a plate width of 166 mm and a plate thickness of 2.5 mm it is found that $A_{pr} = 0.000415$ m^2. The blade is made of steel sheet with a density ρ_{pr} of about $\rho_{pr} = 7.8 * 10^3$ kg/m^3. In figure 4 it can be seen that for high wind speeds, the rotor is running at a tip speed ratio of about $\lambda = 7$. Substitution of $C_t = 0.75$, $\rho = 1.2$ kg/m^3, R = 1.5 m, B = 2, $A_{pr} = 0.000415$ m^2, $\rho_{pr} = 7.8 * 10^3$ kg/m^3 and $\lambda = 7$ in formula 20 gives: ε = 1.15°. This angle is smaller than the calculated angle of 2.33° with which the blade would bend backwards if the compensating effect of the centrifugal moment is not taken into account. This means that the real bending angle will be less than 1.15°.

The real bending angle ε is determined a follows. A thrust moment $M_t = 132.4$ Nm is working backwards and M_t is independent of ε for small values of ε. A bending moment M_b is working forwards and M_b is proportional with ε. $M_b = 132.4$ Nm for ε = 2.33°. A centrifugal moment M_c is working forwards and M_c is also proportional with ε. $M_c = 132.4$ Nm for ε = 1.15°. The path of these three moments is given in figure 9. The sum total of $M_b + M_c$ is determined and the line $M_b + M_c$ is also given in figure 9.

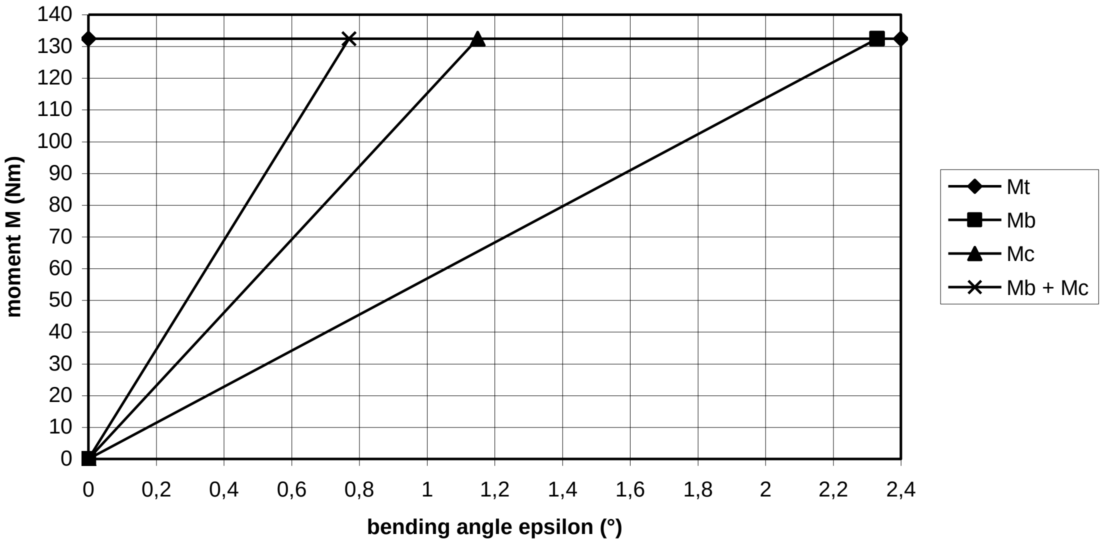

fig. 9 Path of M_t, M_b, M_c, and $M_b + M_c$ as a function of ε

The point of intersection of the line of M_t with the line of $M_b + M_c$ gives the final angle ε. In figure 9 it can be seen that $\varepsilon = 0.77°$. This is a factor 0.33 of the calculated angle of 2.33°. Because the bending stress is proportional to the bending angle it will also be a factor 0.33 of the calculated stress of 155 N/mm^2 resulting in a stress of about 51 N/mm^2. This is a low stress but up to now the gyroscopic moment, which can be rather large, has not yet been taken into account.

The gyroscopic moment is caused by simultaneously rotation of rotor and head. One can distinguish the gyroscopic moment in a blade and the gyroscopic moment which is exerted by the whole rotor on the rotor shaft, and so on the head. On a rotating mass element dm at a radius r, a gyroscopic force dF is working which is maximum if the blade is vertical and zero if the blade is horizontal and which varies with $\sin\alpha$ with respect to a rotating axis frame. α is the angle with the blade axis and the horizon. So it is valid that $dF = dF_{max} * \sin\alpha$. The direction of dF depends on the direction of rotation of both axis and dF is working forwards or backwards. The moment dF * r which is exerted by this force with respect to the blade is therefore varying sinusoidal too.

However, if the moment is determined with respect to a fixed axis frame it can be proven that it varies with $dF_{max} * r \sin^2\alpha$ with respect to the horizontal x-axis and with $dF_{max} * \sin\alpha * \cos\alpha$ with respect to the vertical y-axis. For two and more bladed rotors it can be proven that the resulting moment of all mass elements around the y-axis is zero.

For a single blade and for two bladed rotors, the resulting moment of all mass elements with respect to the x-axis is varying with $\sin^2\alpha$, so just the same as for a single mass element. However, for three and more bladed rotors, the resulting moment of all mass elements with respect to the x-axis is constant. The resulting moment with respect to the x-axis for a three (or more) bladed rotor is given by the formula:

$$M_{gyr\ x\text{-}as} = I_{rot} * \Omega_{rot} * \Omega_{head} \qquad \text{(Nm)} \qquad (23)$$

In this formula I_{rot} is the mass moment of inertia of the whole rotor, Ω_{rot} is the angular velocity of the rotor and Ω_{head} is the angular velocity of the head. The resulting moment is constant for a three bladed rotor because adding three $\sin^2\alpha$ functions which make an angle of 120° which each other, appear to result in a constant value. The resulting moment for a two bladed rotor fluctuates just as it is does for one blade because the moments of both blades are strengthening each other. Formula 23 is still valid for the average value of the moment but the momentary value is given by:

$$M_{gyr\ x\text{-}as} = 2\sin^2\alpha * I_{rot} * \Omega_{rot} * \Omega_{head} \qquad \text{(Nm)} \qquad (24)$$

This function is given in figure 10.

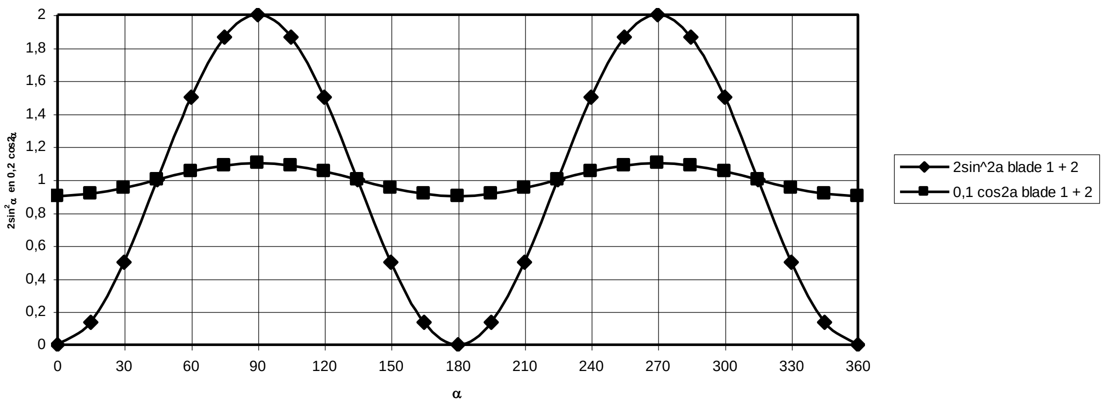

fig. 10 Path of 2 $\sin^2\alpha$ and (1 – 0.1 cos2α) for a two bladed rotor

Formula 24 can also be written as:

$$M_{gyr\ x\text{-}as} = (1 - \cos 2\alpha) * I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (25)$$

Because the average of $\cos 2\alpha$ is zero, the average of formula 25 is the same as formula 23.

Up to now it is assumed that the blades have an infinitive stiffness. However, in reality the blades are very flexible and will bend by the fluctuations of the gyroscopic moment. Therefore the blade will not follow the curve for which formula 24 and 25 are valid. I am not able to describe this effect physically but the practical result of it is that the strong fluctuation on the 2 $\sin^2\alpha$ function is almost flattened. However, the average moment is assumed to stay the same as given by formula 23. I estimate that the flattened curve can be given by:

$$M_{gyr\ x\text{-}as\ flattened} = (1 - 0.1 \cos 2\alpha) * I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (26)$$

The function $(1 - 0.1 \cos 2\alpha)$ is also plotted in figure 10. This function has a maximum for $\alpha = 90°$ and for $\alpha = 270°$. The maximum is $1.1 * I_{rot} * \Omega_{rot} * \Omega_{head}$.

For the calculation of the blade strength we are not interested in the variation of the gyroscopic moment with respect to a fixed axis frame but in variation of the moment in the blade itself so with respect to a rotation axis frame for which it was explained earlier that the moment is varying sinusoidal. If the blade is vertical both axis frames coincide and the moment for both axis frames is the same. The maximum moment in one blade is then halve the value of the moment for the whole rotor.

Therefore the maximum moment in one blade is given by:

$$M_{gyr\ bl\ max} = 0.55 * I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (27)$$

For a two bladed rotor, the moment of inertia of the whole rotor I_{rot} is twice the moment of inertia of one blade I_{bl}. Therefore it is valid that:

$$M_{gyr\ bl\ max} = 1.1\ I_{bl} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (28)$$

For the chosen blade geometry it is calculated that $I_{bl} = 3.84$ kgm^2. The maximum loaded rotational speed of the rotor can be read in figure 4 and it is found that $n_{max} = 410$ rpm. This gives $\Omega_{rot\ max} = 42.9$ rad/s (because $\Omega = \pi * n / 30$).

It is not easy to determine the maximum yawing speed. The VIRYA-3B2 is provided with the hinged side vane safety system which has a light vane blade and a large moment of inertia of the whole head around the tower axis. This is because the vane arm is a part of the head. For sudden variations in wind speed and wind direction the vane blade will therefore react very fast but the head will follow only slowly. It is assumed that the maximum angular velocity of the head can be 0.2 rad/s at very high wind speeds.

Substitution of $I_{bl} = 3.84$ kgm^2, $\Omega_{rot\ max} = 42.9$ rad/s en $\Omega_{head\ max} = 0.2$ rad/s in formula 28 gives: $M_{gyr\ bl\ max} = 36.2$ Nm = 36200 Nmm.

Substitution of M = 36200 Nmm, b = 80 mm and h = 8 mm in formula 16 gives $\sigma_{b\ max} = 43$ N/mm^2. This value has to be added to the bending stress of 51 N/mm^2 which was the result of the thrust because there is always a position were both moments are strengthening each other. This gives $\sigma_{b\ tot\ max} = 94$ N/mm^2. The minimum stress is 51 – 43 = 8 N/mm^2. So the stress is always positive and therefore it is probably not necessary to take the load as a fatigue load. For the strip material hot rolled strip Fe 360 (MCB quality S235JRG2) is chosen. The 0.2 % deformation limit for hot rolled strip with a thickness less than 16 mm is given as 235 N/mm^2. However this is for a tensile stress. The 0.2 % deformation limit for a bending stress is much higher and it is assumed that it is about 300 N/mm^2.

It is assumed that the allowable bending stress for a fatigue load is 200 N/mm^2 and for a non fatigue load is 260 N/mm^2. The calculated stress is much lower so the strip is strong enough, even if the load is taken as a fatigue load.

In reality the blade is not extremely stiff and will also bend. This reduces the bending of the strip and therefore the real stress in the strip will be lower.

8.2 Bending stress in the strip for a slowed down rotor

The rotational speed for a rotor which is slowed down by making short-circuit of the generator is very low. Therefore there is no compensating effect of the centrifugal moment on the moment of the thrust. However, there is also no gyroscopic moment. The safety system is also working if the rotor is slowed down but a much larger wind speed will be required to generate the same thrust as for a rotating rotor.

In chapter 6.1 it has been calculated that the maximum thrust on one blade for a rotating rotor is 129 N for $V = V_{rated} = 11$ m/s and $\delta = 35°$. The head turns out of the wind such at higher wind speeds, that the thrust stays almost constant above V_{rated}. A slowed down rotor will therefore also turn out of the wind by 35° if the force on one blade is 129 N. Also for a slowed down rotor the force is staying constant for higher yaw angles. However, for a slowed down rotor, the resulting force of the blade load is exerting in the middle of the blade at $r_4 = 0.875$ m because the relative wind speed is almost constant along the whole blade. The bending moment around the edge of the hub is therefore somewhat smaller. Formula 13 changes into:

$$M_{b\,t} = F_{t\,\delta\,bl} * (r_4 - r_2) \qquad \text{(Nm)} \qquad (27)$$

Substitution of $F_{t\,\delta\,bl} = 129$ N, $r_4 = 0.875$ m en $r_2 = 0.024$ m in formula 27 gives $M_{b\,t} = 110$ Nm = 110000 Nmm. Substitution of M = 110000 Nmm, b = 80 mm and h = 8 mm in formula 14 gives $\sigma_b = 129$ N/mm^2. This is higher than the calculated stress for a rotating rotor. However, this load is not fluctuating and therefore it is surly not necessary to use the allowable fatigue stress. The allowable bending stress is about 260 N/mm^2 for hot rolled strip Fe 360, so the strip is strong enough.

Because the strip and the blade are rather flexible it has to be checked if a slowed down rotor can't hit the tower. In chapter 6.1 it has been calculated, for no compensation of the gyroscopic moment, that the bending angle is 2.33° for a stress of 155 N/mm^2. So for a stress of 129 N/mm^2 the bending angle will be 2.33 * 129 / 155 = 1.94°. For a rotor radius of R = 1.5 m this results in a movement at the tip of about 0.051 m. Because the blade itself will bend too, the movement will be larger and it is expected that it will be about 0.07 m. The minimum distance in between the blade tip and the tower pipe is about 0.28 m if the blade is not bending. So there is no chance that the blade hits the tower for a slowed down rotor.

9 Hub for generator with frame size 100 and original motor shaft

The alternative generator with frame size 100 and the original motor shaft is described in report KD 503 (ref. 8). The original motor shaft has a shaft end with a diameter of 28 mm and a length of 60 mm. It is provided with a key with a width of 8 mm. The distance in between the top of the key and the opposite side of the shaft is 31 mm. A hub is designed which is clamped onto this shaft and locked by the key and by a central screw M10 * 20 mm. For this hub, the width of the strip which connects both blades must be at least 80 mm and this is the reason why this report KD 467 has been modified in September 2012 (the first version of September 2011 uses a strip width of 60 mm).

The hub is made of two mild steel strips size 60 * 20 * 80 mm. One can use strip size 60 * 20 mm or size 80 * 20 mm. The strips are clamped together at a distance of 8 mm using four bolts M8 * 60 mm and a 28 mm hole is turned in the centre. This hole is used for clamping the hub around the original motor shaft. The two upper bolts M8 * 60 mm prevent that the key is lost.

The central strip is bolted to the front side of the two hub halves by four screws M8 * 20 mm. Two threaded holes have to be made in the front side of each hub halve. First it was thought to use four bolts M8 * 80 mm and four self locking nuts M8 but the bolt ends will probably touch the front side of the generator housing and therefore this idea was cancelled. The original motor shaft is normally provided with a central threaded hole M10 and a screw M10 * 20 mm is used to realise axial fixation of the hub. A picture of the hub is given in appendix 2.

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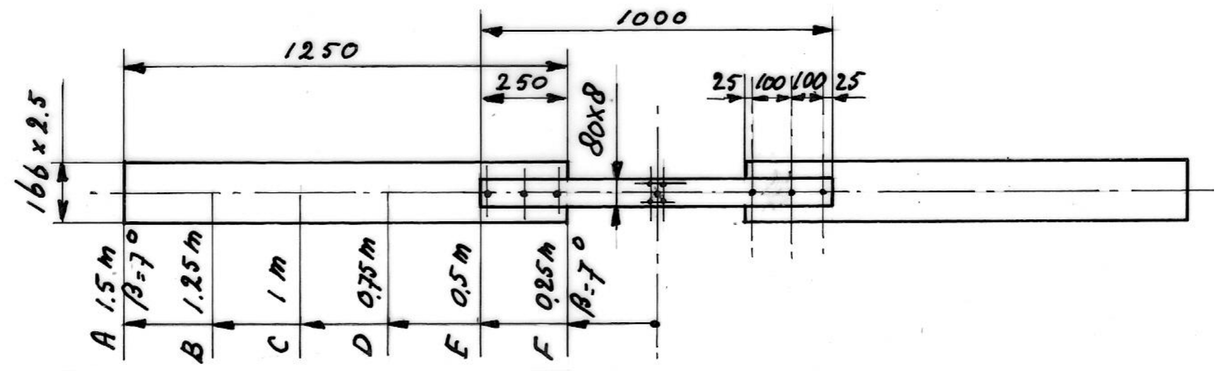
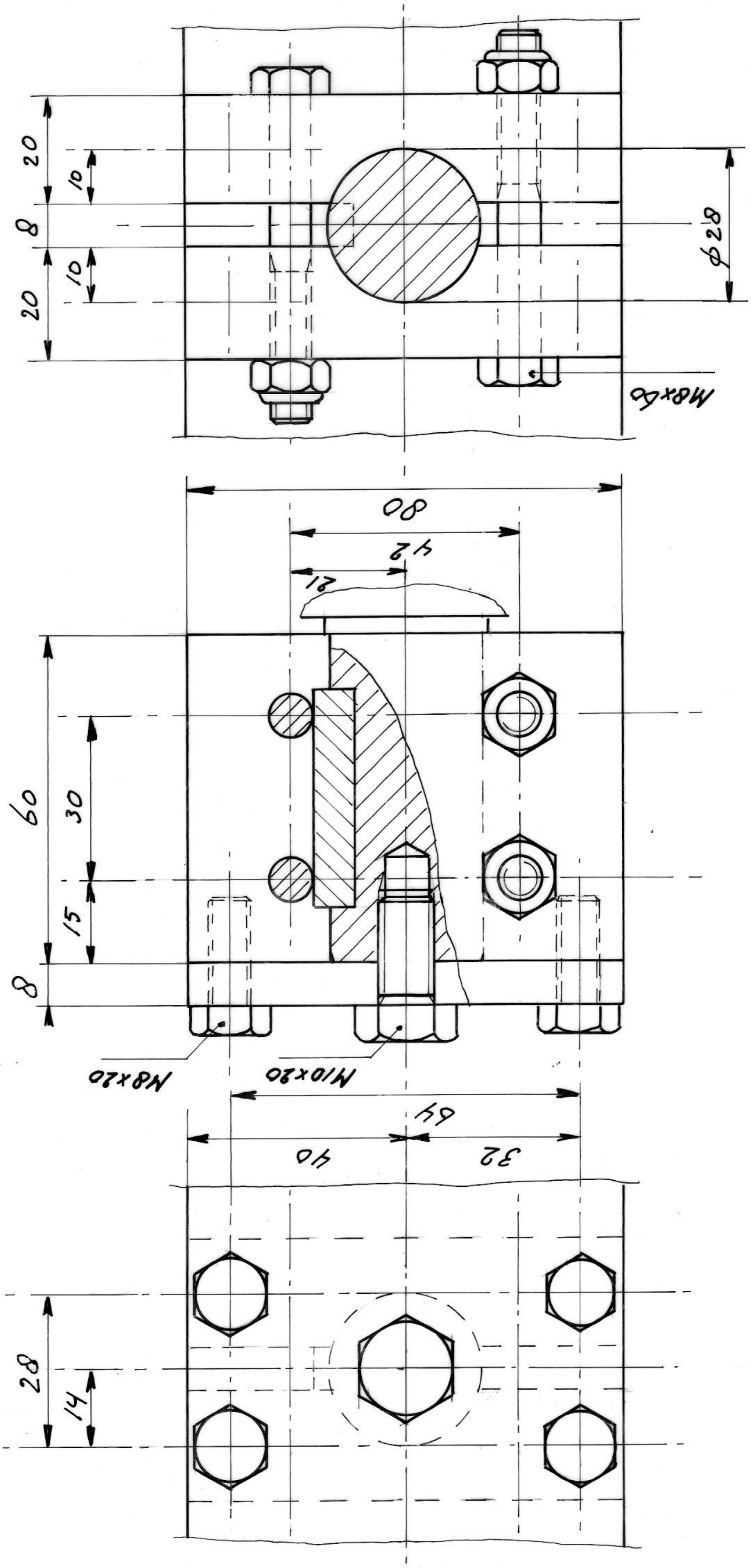
Appendix 1



Sketch VIRYA-3B2 rotor

Appendix 2



Sketch hub for generator frame size 100 and original motor shaft