Development of a pendulum safety system with a torsion spring and e = 0.2 R

ing. A. Kragten

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engineering office Kragten Design
Populierenlaan 51
5492 SG Sint-Oedenrode
The Netherlands
telephone: +31 413 475770
e-mail: info@kdwindturbines.nl
website: www.kdwindturbines.nl

Contains page

1 Introduction

Windmills with fixed rotors can be protected against too high forces and too high rotational speeds by turning the rotor out of the wind. This can be done around a vertical and around a horizontal axis. All present VIRYA windmills developed by Kragten Design turn out of the wind around a vertical axis and make use of the so called hinged side vane safety system.

Safety systems for water pumping windmills are described in report R 999 D (ref. 1). Water pumping windmills normally have fixed rotors and all safety systems are working by turning the rotor out of the wind. However, electricity generating windmills can also be protected by turning the rotor out of the wind. In chapter 2 of R 999 D, the reasons are given why a safety system is necessary. These reasons are:

1 Limitation of the axial force or thrust on the rotor to limit the load on the rotor blades, the tower and the foundation.
2 Limitation of the rotational speed of the rotor to limit the centrifugal force in the blades, imbalance forces, high gyroscopic moments in the blades and the rotor shaft, to prevent flutter for blades with low torsion stiffness and to prevent too high rotational speeds of the load which is relevant for limitation of heat dissipation in a generator or for limitation of shock forces in the transmission to a piston pump.
3 Limitation of the yawing speed to limit high gyroscopic moments in the blades and the rotor shaft.

Almost all known systems are described shortly in chapters 3 and 4 of R 999 D. The three most generally used systems, the ecliptic system, the inclined hinge main vane system and the hinged side vane system, are described in detail in chapter 7 of report R 999 D. Because report R 999 D is no longer available, the hinged side vane system is also described in several KD-reports. It is described in report KD 213 (ref. 2) for the VIRYA-4.2 windmill. The ecliptic system with a torsion spring is described in report KD 409 (ref. 3). The inclined hinge main vane system is described in report KD 431 (ref. 4)

A safety system for which the rotor turns out of the wind around an horizontal axis is described in report KD 377 ”Development of a tornado proof pendulum safety system for a medium size wind turbine which turns the rotor out of the wind along an horizontal axis” (ref. 5). The hinge axis for this system is positioned in the rotor plane and at a distance of about 1.1 R from the rotor axis. The counterbalancing moment is provided by two heavy weights which are positioned on long arms which move along the tower pipe.

Any safety system has certain advantages and certain disadvantages. The main disadvantage of the pendulum system which is described in report KD 377, is that the total weight of rotor, generator and safety system is rather large. In chapter 6 of KD 377 some alternatives are given. One of the alternatives to reduce the weight, is to position the hinge axis behind the rotor plane and to reduce the eccentricity in between the hinge axis and the rotor axis. However, this alternative has as disadvantage that now the system must have a stop, otherwise the rotor will touch the tower pipe at low wind speeds. The stop must be elastic or the movement of the rotor must be damped, otherwise large shock forces will occur if the stop is hit. But reduction of the total weight is important and the system might be more attractive than the original pendulum safety system with very large eccentricity. Another way to reduce the weight is to use a torsion spring in stead of balancing weights. The use of a torsion spring has also as advantage that the ideal δ-V curve (see chapter 2) can be approached rather close. A system with a torsion spring is described in report KD 438 (ref. 6). For this system the eccentricity e has been taken rather large (e = 0.4 R) and then it is allowed to neglect the influence of the self orientating moment M_{so} on the rotor moment M_{rotor}. However, for this large eccentricity the whole construction looks rather ugly and a strong torsion spring is required. So the system is now investigated for a smaller eccentricity e = 0.2 R in this report KD 439. But for this smaller eccentricity the self orientating moment can no longer be neglected. The system is described in chapter 3.

2 The ideal δ-V curve

Generally it is wanted that the windmill rotor is perpendicular to the wind up to the rated wind speed V_{rated}, and that the rotor turns out of the wind such that the rotational speed, the rotor thrust, the torque and the power stay constant above V_{rated}. It appears to be that the component of the wind speed perpendicular to the rotor plane determines these four quantities. The yaw angle δ is the angle in between the wind direction and the rotor axis. The component of the wind speed perpendicular to the rotor plane is therefore V cosδ. The formulas for a yawing rotor for the rotational speed n_δ, the rotor thrust $F_{t\delta}$, the torque Q_δ and the power P_δ are given in chapter 7 of report KD 35 (ref. 7). These formulas are copied as formula 1, 2, 3 and 4.

$$n_\delta = 30 * \lambda * \cos\delta * V / \pi R \qquad \text{(rpm)} \tag{1}$$

$$F_{t\delta} = C_t * \cos^2\delta * ½\rho V^2 * \pi R^2 \qquad \text{(N)} \tag{2}$$

$$Q_\delta = C_q * \cos^2\delta * ½\rho V^2 * \pi R^3 \qquad \text{(Nm)} \tag{3}$$

$$P_\delta = C_p * \cos^3\delta * ½\rho V^3 * \pi R^2 \qquad \text{(W)} \tag{4}$$

These four quantities stay constant above V_{rated} if the component of the wind speed perpendicular to the rotor plane is kept constant above V_{rated}. So in formula:

$$V \cos\delta = V_{rated} \qquad (\text{for } V > V_{rated}) \tag{5}$$

It is assumed that the rotor is loaded such that it runs at the design tip speed ratio λ_d. If the wind speed is in between 0 m/s and V_{rated}, the n-V curve is a straight line through the origin. The F_t-V and the Q-V curves are then parabolic lines and the P-n curve is a cubic line.

Formula 5 can be written as:

$$\delta = \arccos (V_{rated} / V) \qquad (°) \tag{6}$$

This formula is given as a graph in figure 1 for different value of V / V_{rated}. The value of δ has been calculated for V / V_{rated} is respectively 1, 1.01, 1.05, 1.1, 1.25, 1.5, 2, 2.5, 3, 4, 5 and 6.

The rated wind speed V_{rated} is chosen on the basis of the maximum thrust and the maximum rotational speed which is allowed for a certain rotor and a certain generator. Mostly V_{rated} is chosen about 10 m/s. For the chosen value of V_{rated}, figure 1 can be transformed into the δ-V curve for which V (in m/s) is given on the x-axis. If it is chosen that V_{rated} = 10 m/s, figure 1 becomes the δ-V curve if all values on the x-axis are multiplied by a factor 10.

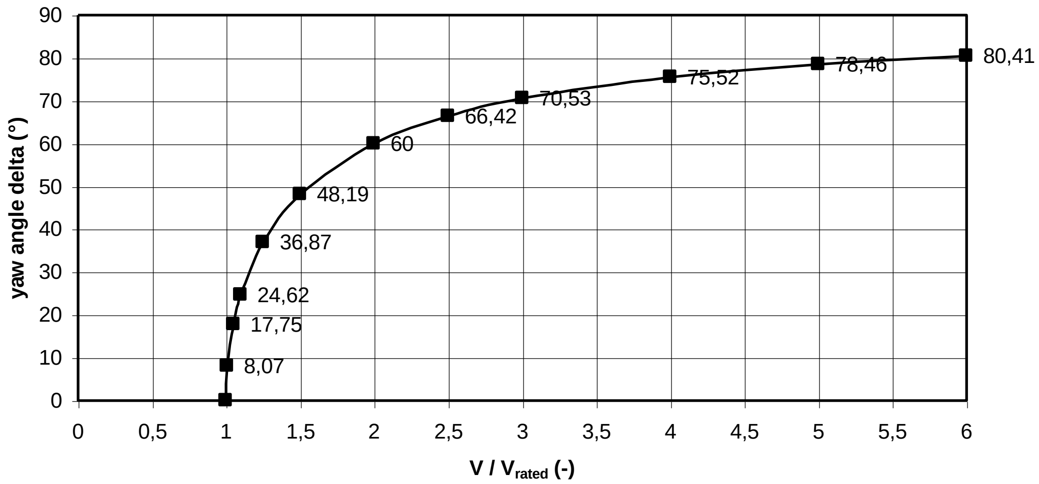

fig. 1 the δ-V/V_{rated} curve for the ideal safety system

In figure 1 it can be seen that the rotor is perpendicular to the wind for (V / V_{rated}) < 1 but that the required change in δ is very sudden if V / V_{rated} is a very little higher than 1. So even if one would have a safety system which theoretically has the ideal δ-V curve, in practice this curve will not be followed because the inertia of the system prevents sudden changes of δ around V / V_{rated} = 1. So the system will turn out of the wind less than according to the ideal δ-V curve. This will result in a certain overshoot of the rotational speed and the thrust. However, the overshoot of the rotational speed is also determined by the moment of inertia of the rotor. Dynamic description of the system is therefore very complicated.

For high values of V / V_{rated}, a certain increase of V, and therefore a certain increase of V / V_{rated}, requires only a relatively small increase of δ. It is therefore much easier to follow the theoretical δ-V curve at high wind speeds.

3 Description of the pendulum safety system with a torsion spring and a rotor with a Gö 623 airfoil

The safety system is called the pendulum safety system because the whole assembly of rotor, generator and beam is swinging on top of the tower like the pendulum of a clock. The horizontal hinge axis is intersecting with the tower axis. For the eccentricity e in between the rotor axis and the hinge axis is chosen that e = 0.2 R. This is rather small if compared to the original pendulum safety system as described in report KD 377 but the same as for the VIRYA-4.2 which is equipped with the hinged side vane system. For e = 0.2 R it is no longer allowed to neglect the contribution of the self orientating moment M_{so} to the rotor moment M_{rotor}. However, it is assumed that the contribution of the side force on the rotor $F_{s\delta}$ can be neglected. So it is assumed that M_{rotor} is only determined by the rotor thrust $F_{t\delta}$ and the eccentricity e and by M_{so}. M_{rotor} is given by:

$$M_{rotor} = F_{t\delta} * e - M_{so} \qquad \text{(Nm)} \qquad (7)$$

In KD 35 no formula is given for the self orientating moment M_{so}. M_{so} is created because the exertion point of the thrust doesn’t coincide with the hart of the rotor. There is only little known about M_{so} and only some very rough measurements have been performed which are given in report R 344 D (in Dutch, ref. 8). For these measurement an unloaded two bladed rotor was used with a design tip speed ratio of 5 and provided with a curved sheet airfoil.

Practical experience with the VIRYA windmills using a Gö 623 airfoil, indicate that M_{so} is much lower for this airfoil. Recently I have made a model of a two bladed rotor with a diameter of 0.8 m with a design tip speed ratio of about 6.5 and using a Gö 623 airfoil. The maximum eccentricity which was possible for which the rotor doesn't turn out of the wind completely, was about 0.027 m. From this measurement it is derived that the maximum self orientating moment for a certain wind speed is about half the value as for the same diameter rotor with a curved sheet airfoil.

M_{so} is given by:

$M_{so} = C_{so} * ½\rho V^2 * \pi R^3$ (Nm) (8)

C_{so} depends on the yaw angle δ and appears to have a maximum for δ = 30°. The estimated C_{so}-δ curve for a rotor with a Gö airfoil with a flat lower side can be approximated by two goniometrical functions, one function for 0° < δ < 40° and one function for 40° < δ < 90°. These functions are:

$C_{so} = 0.0225 \sin 3\delta$ (-) (for 0° < δ < 40°) (9)

$C_{so} = 0.0332 \cos^2\delta$ (-) (for 40° < δ < 90°) (10)

If the direction of the moment for a negative value of δ is taken the same as for a positive value of δ, formula 9 can also be used for –40° < δ < 0°. The path of both curves is given in figure 2.

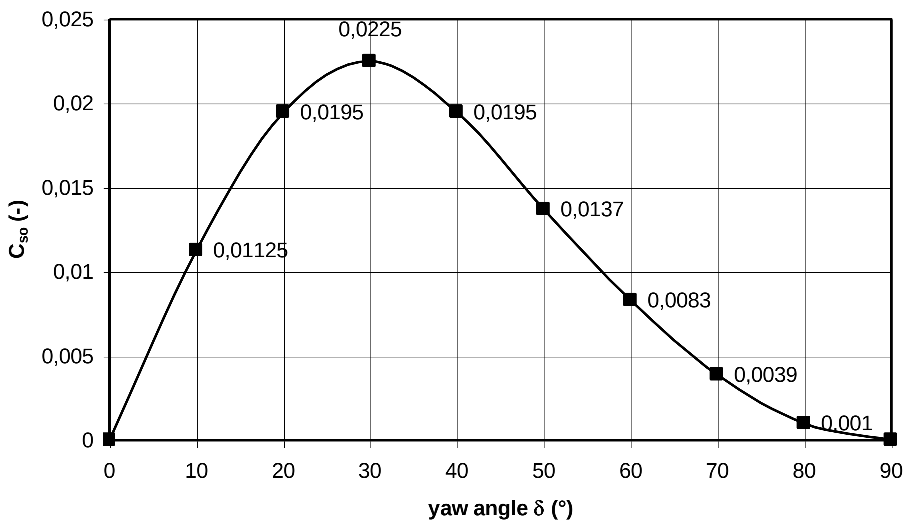

fig. 2 Path of C_{so} as a function of the yaw angle δ

(8) + (9) gives:

$M_{so} = 0.0225 \sin 3\delta * ½\rho V^2 * \pi R^3$ (Nm) (for 0° < δ < 40°) (11)

(8) + (10) gives:

$M_{so} = 0.0332 \cos^2\delta * ½\rho V^2 * \pi R^3$ (Nm) (for 40° < δ < 90°) (12)

(2) + (7) + (11) and e = 0.2 R gives:

$M_{rotor} = 0.2 * C_t * \cos^2\delta * ½\rho V^2 * \pi R^3 - 0.0225 \sin 3\delta * ½\rho V^2 * \pi R^3$ or
$M_{rotor} = ½\rho V^2 * \pi R^3 (0.2 * C_t * \cos^2\delta - 0.0225 \sin 3\delta)$ (Nm)
(for 0° < δ < 40°) (Nm) (13)

(2) + (7) + (12) and e = 0.2 R gives:

$M_{rotor} = 0.2 * C_t * \cos^2\delta * ½\rho V^2 * \pi R^3 - 0.0332 \cos^2\delta * ½\rho V^2 * \pi R^3$ or
$M_{rotor} = ½\rho V^2 * \pi R^3 * \cos^2\delta (0.2 * C_t - 0.0332)$ (Nm)
(for 40° < δ < 90°) (Nm) (14)

Apart from the aerodynamic moment M_{rotor}, there is also a moment working around the hinge axis which is caused by the weight of the rotor, the generator and the swinging parts of the head. All these parts together result in a total weight of the swinging parts G (in N), acting at the centre of gravity which is lying at a certain radius r_G from the hinge axis. The position of the centre of gravity is lying a bit below the rotor axis because of the beam which connects the generator to the horizontal axis bearing housing. The head geometry is chosen such that angle α_0 in between r_G and the rotor plane is 30°. The right hand angle in between r_G and the vertical plane is called α. The right hand angle in between the rotor plane and the vertical plane is called δ (see figure 3). Figure 3 is drawn for a yaw angle δ = 50° belonging to a wind speed V = 14.82 m/s (see table 1).

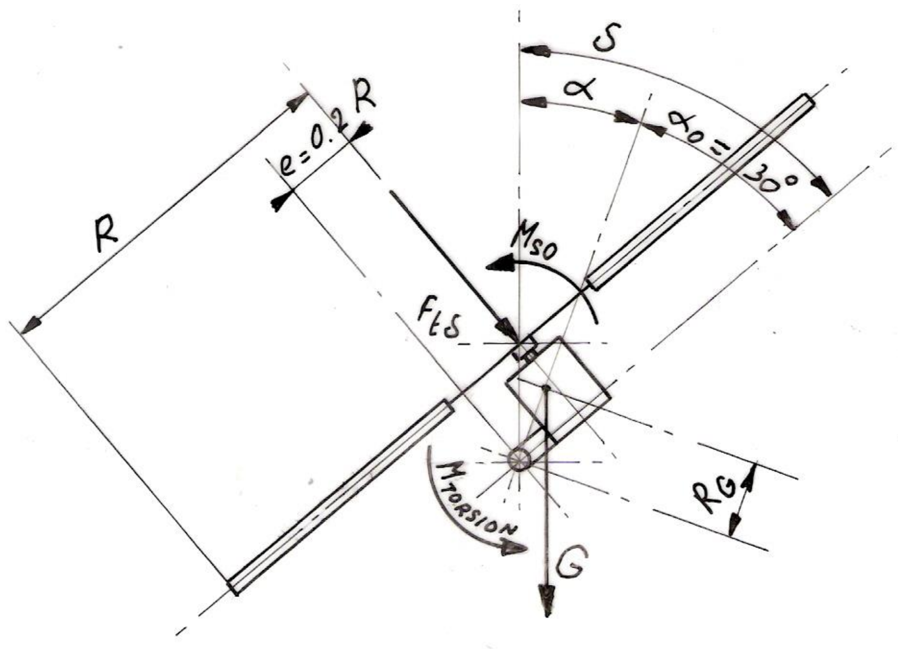

fig. 3 Side view of the pendulum safety system with a torsion spring for δ = 50 °

The relation in between α, δ and α_0 is given by:

$\alpha = \delta - \alpha_0$ (°) (15)

So α = -30° for δ = 0° and α_0 = 30°. For δ = 0°, the left hand moment M_G produced by G around the hinge axis is taken positive. The left hand moment M_G is therefore given by:

$M_G = G * R_G * \sin(-\alpha)$ (Nm) (16)

(15) + (16) and $\alpha_0 = 30°$ gives:

$M_G = G * R_G * \sin(30° – \delta)$ (Nm) (17)

M_G has an extreme value $M_{G\ max}$ for $\alpha = 90°$ and for $\alpha = -90°$, so for $\delta = 120°$ and for $\delta = -60°$. So $M_{G\ max}$ is given by:

$M_{G\ max} = G * R_G$ (Nm) (18)

So it is valid that:

$M_G / M_{G\ max} = \sin(30° – \delta)$ (-) (19)

This function is given in figure 4 for $0° < \delta < 90°$.

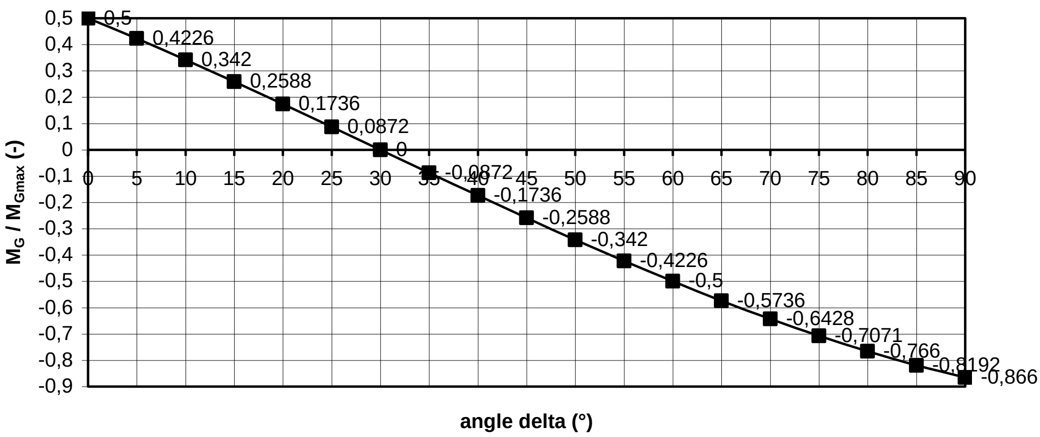

fig. 4 $M_G / M_{G\ max}$ as a function of δ for $\alpha_0 = 30°$

In figure 4 it can be seen that the M_G / M_{Gmax} – α curve is about a straight line for $0° < \delta < 60°$.

Apart from M_{rotor} and M_G there is also working a left hand moment $M_{torsion}$ around the hinge axis caused by the torsion spring. The torsion spring is chosen such that $M_{torsion} = 0$ for $\delta = 0°$. The torsion spring is also chosen such that $M_{torsion} = M_{Gmax}$ for $\delta = 65°$. $M_{torsion}$ increases linear to δ, so $M_{torsion}$ is given by:

$M_{torsion} = G * R_G * \delta/65°$ (Nm) (20)

The ratio $M_{torsion}$ / M_{Gmax} as a function of δ is given in figure 5.

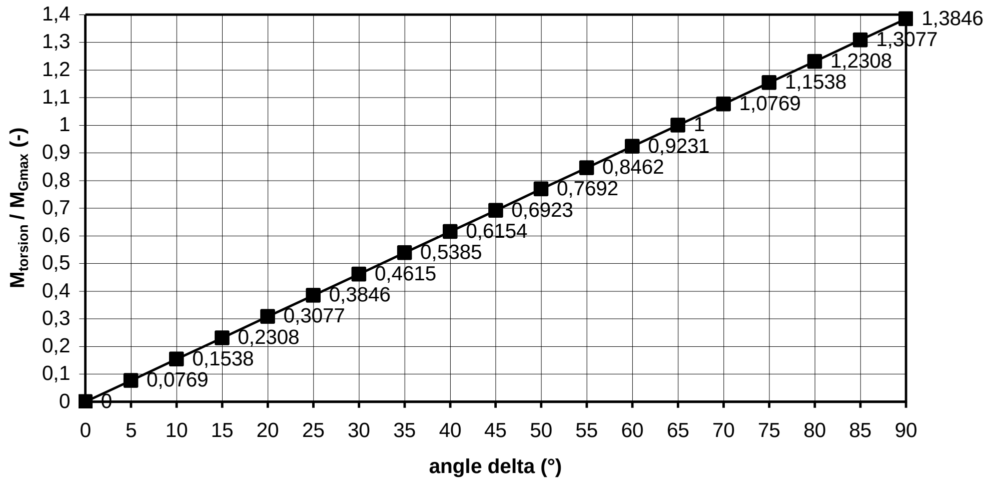

fig. 5 $M_{torsion}$ / $M_{G\ max}$ as a function of δ

The total effect of M_G / M_{Gmax} + $M_{torsion}$ / M_{Gmax} can be shown by adding the curves of figure 4 and figure 5. This results in figure 6.

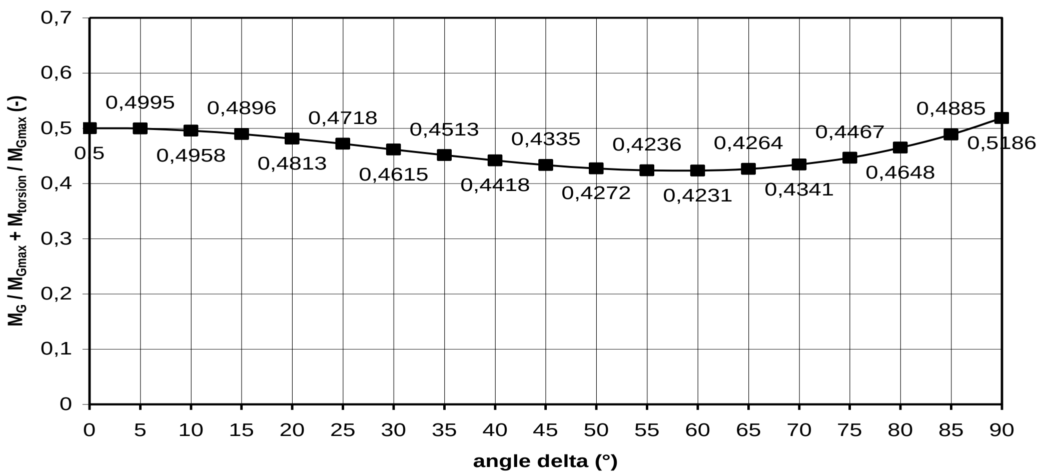

fig. 6 M_G / M_{Gmax} + $M_{torsion}$ / M_{Gmax} as a function of δ

In figure 6 it can be seen that the resulting moment is a little decreasing for 0° < δ < 60°. This decreasing partly compensates the self orientating moment. The decreasing curve prevents that the maximum rotational speed and thrust at high wind speeds are too high.

For a quasi-stationary situation there is balance of moments in between the right hand moment M_{rotor} and the left hand moments M_G and $M_{torsion}$. The moment equation is given by:

$$M_{rotor} = M_G + M_{torsion} \tag{21}$$

(13) + (17) + (20) + (21) gives:

$\frac{1}{2}\rho V^2 * \pi R^3 * (0.2 * C_t * \cos^2\delta - 0.0225 \sin 3\delta) = G * R_G * \{\sin(30° – \delta) + \delta/65°\}$ or
$\frac{1}{2}\rho V^2 * (0.2 * C_t * \cos^2\delta - 0.0225 \sin 3\delta) * \pi R^3 / (G * R_G) = \sin(30° – \delta) + \delta/65°$
(for $0° < \delta < 40°$) (22)

(14) + (17) + (20) + (21) gives:

$\frac{1}{2}\rho V^2 * \pi R^3 \cos^2\delta * (0.2 * C_t - 0.0332) = G * R_G * \{\sin(30° – \delta) + \delta/65°\}$ or
$\frac{1}{2}\rho V^2 * \cos^2\delta * (0.2 * C_t - 0.0332) * \pi R^3 / (G * R_G) = \sin(30° – \delta) + \delta/65°$
(for $40° < \delta < 90°$) (23)

The design wind speed V_d is defined as the wind speed for which the head just starts moving backwards but for which δ is still just 0°. It is assumed that all parameters of the safety system are chosen such that V_d = 9 m/s. For $V = V_d$ and $\delta = 0°$ formula 22 changes into:

$0.2 * C_t * \frac{1}{2}\rho V_d^2 * \pi R^3 / (G * R_G) = \sin 30°$ or

$R^3 / (G * R_G) = \sin 30° / (0.2 * C_t * \pi * \frac{1}{2}\rho V_d^2)$ (24)

The thrust coefficient is about 0.7 for a rotor running at the design tip speed ratio. The air density ρ is about 1.2 kg/m^3 for air of 20 °C at sea level. It is chosen that V_d = 9 m/s. Substitution of these values in formula 24 gives:

$R^3 / (G * R_G) = 0.0234$ (for V_d = 9 m/s) (25)

The rotor and head geometry has to be chosen such that equation 25 is fulfilled.

(22) + (25) gives:

$0.0234 * \frac{1}{2}\rho V^2 * (0.2 * C_t * \cos^2\delta - 0.0225 \sin 3\delta) * \pi = \sin(30° – \delta) + \delta/65°$
(for $0° < \delta < 40°$) (26)

Substitution of ρ = 1.2 kg/m^3 and C_t = 0.7 (-) in formula 26 gives:

$0.0441 * V^2 * (0.14 * \cos^2\delta - 0.0225 \sin 3\delta) = \sin(30° – \delta) + \delta/65°$ or

$V = \sqrt{[\{\sin(30° – \delta) + \delta/65°\} / \{0.0441 * (0.14 * \cos^2\delta - 0.0225 \sin 3\delta)\}]}$
(for $0° < \delta < 40°$) (27)

(23) + (25) gives:

$0.0234 * \frac{1}{2}\rho V^2 * \cos^2\delta * (0.2 * C_t - 0.0332) * \pi = \sin(30° – \delta) + \delta/65°$
(for $40° < \delta < 90°$) (28)

Substitution of ρ = 1.2 kg/m^3 and C_t = 0.7 (-) in formula 28 gives:

$0.0441 * V^2 * \cos^2\delta * (0.14 - 0.0332) = \sin(30° – \delta) + \delta/65°$ or
$0.00471 * V^2 * \cos^2\delta = \sin(30° – \delta) + \delta/65°$ or

$V = \sqrt{[\{\sin(30° – \delta) + \delta/65°\} / (0.00471 * \cos^2\delta)]}$ (for $40° < \delta < 90°$) (29)

Next formula 27 and 29 are used to calculate V for different values of δ. It has been chosen that δ = 0°, 10°, 20°, 30°, 40°, 50°, 60°, 70° and 80°. The result is given in table 1.

δ (°)	V (m/s)	cosδ	$\cos^2\delta$	$\cos^3\delta$	V * cosδ	$V^2 * \cos^2\delta$	$V^3 * \cos^3\delta$
0	9	1	1	1	9	81	729
10	9.5023	0.9848	0.9698	0.9551	9.358	87.567	819.474
20	10.2377	0.9397	0.8830	0.8298	9.620	92.548	890.391
30	11.2631	0.8660	0.75	0.6495	9.754	95.143	928.011
40	12.6420	0.7660	0.5868	0.4495	9.684	93.799	908.191
50	14.8164	0.6428	0.4132	0.2656	9.524	90.708	862.137
60	18.9552	0.5	0.25	0.125	9.478	89.825	851.324
70	28.0705	0.3420	0.1170	0.0400	9.600	92.190	884.729
80	57.2028	0.1736	0.0302	0.00524	9.930	98.819	980.806

table 1 Calculated relation in between δ and V for V_d = 9 m/s

In table 1 the values for cosδ, $\cos^2\delta$, $\cos^3\delta$, V * cosδ, $V^2 * \cos^2\delta$ and $V^3 * \cos^3\delta$ are also mentioned. V * cosδ is an indication for the increase of the rotational speed (see formula 1). $V^2 * \cos^2\delta$ is an indication for the increase of the thrust and the torque (see formula 2 and 3). $V^3 * \cos^3\delta$ is an indication for the increase of the power (see formula 4). The values for δ as a function of V are given as the δ-V curve figure 7.

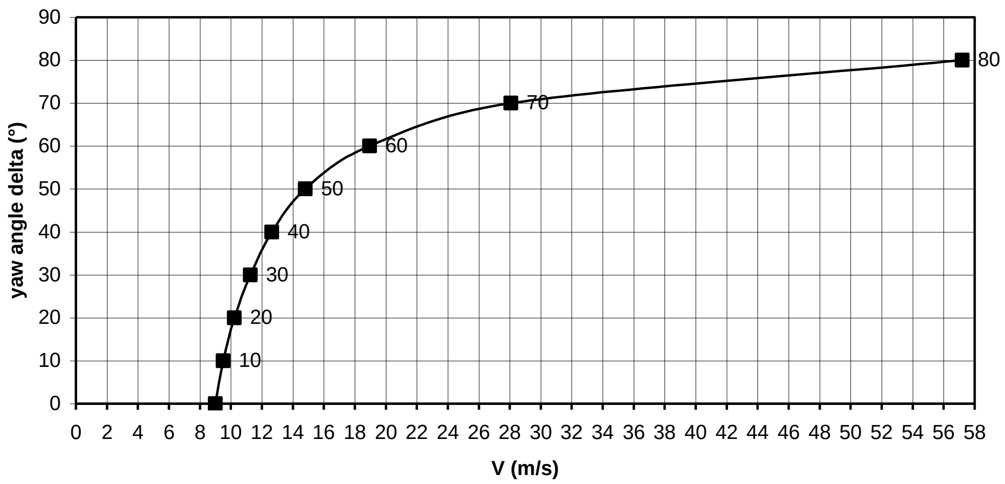

fig. 7 Calculated δ-V curve for the pendulum safety system with a torsion spring and a Gö 623 airfoil for V_d = 9 m/s

In table 1 and figure 7 it can be seen that a very large increase of the wind speed (from 28.0705 m/s up to 57.2028 m/s) is needed to increase the yaw angle from 70° to 80°. The helicopter position (for δ = 90°) will never be reached for stationary conditions. However, hard wind gusts at hurricanes or tornados will cause swinging movements resulting in reaching δ = 90° for a short moment. To make the system tornado proof, it must be possible to fix the movement at δ = 90°, so there must be a stop and a kind of clamp at this angle. This clamp can work automatically and it will lock the rotor at δ = 90°. If this has happened, one has to unlock the system manually but as automatic locking will happen only during very high wind gusts, automatic locking is an acceptable option. It must be possible to turn the rotor to the helicopter position and to unlock it while standing on the tower at about 2/3 of the height.

High upwards wind speeds may occur in the central part of a tornado, so the rotor may start rotating even if the safety system is locked in the helicopter position. However, the rotor will rotate backwards if the wind comes from below and the airfoil will therefore have a lot of drag which strongly limits the rotational speed and the thrust. So the rotor will survive large upwards wind speeds. But there may be extremely strong tornados which will not be survived.

The calculated values of V * cosδ as a function of V are given in figure 8. This curve is an indication of the variation of the rotational speed for $V > V_d$.

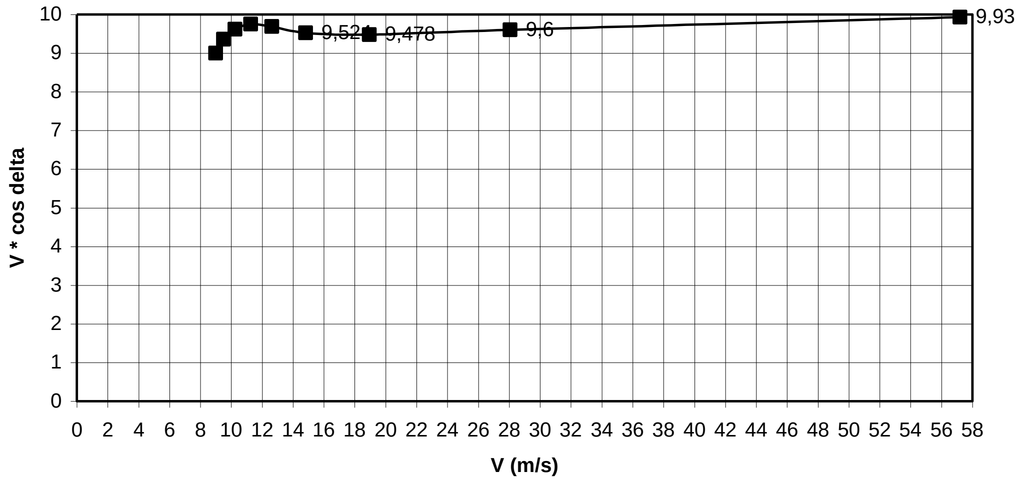

fig. 8 V * cosδ as a function of V for the pendulum safety system with a torsion spring and a Gö 623 airfoil for V_d = 9 m/s

In figure 8 it can be seen that the rotational speed is sharply limited and almost constant for 10 < V < 58 m/s.

The calculated values of $V^2 * \cos^2\delta$ as a function of V are given in figure 9. This curve is an indication of the variation of the thrust and the torque for $V > V_d$.

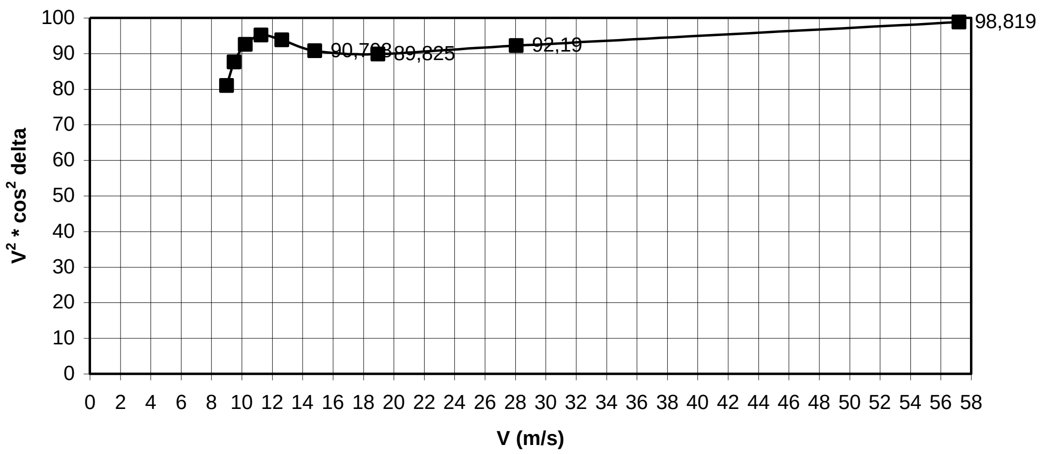

fig. 9 $V^2 * \cos^2\delta$ as a function of V for the pendulum safety system with a torsion spring and a Gö 623 airfoil for V_d = 9 m/s

In figure 9 it can be seen that the thrust and the torque is almost constant for 10 < V < 58 m/s.

The calculated values of $V^3 * \cos^3\delta$ as a function of V are given in figure 10. This curve is an indication of the variation of the power for $V > V_d$.

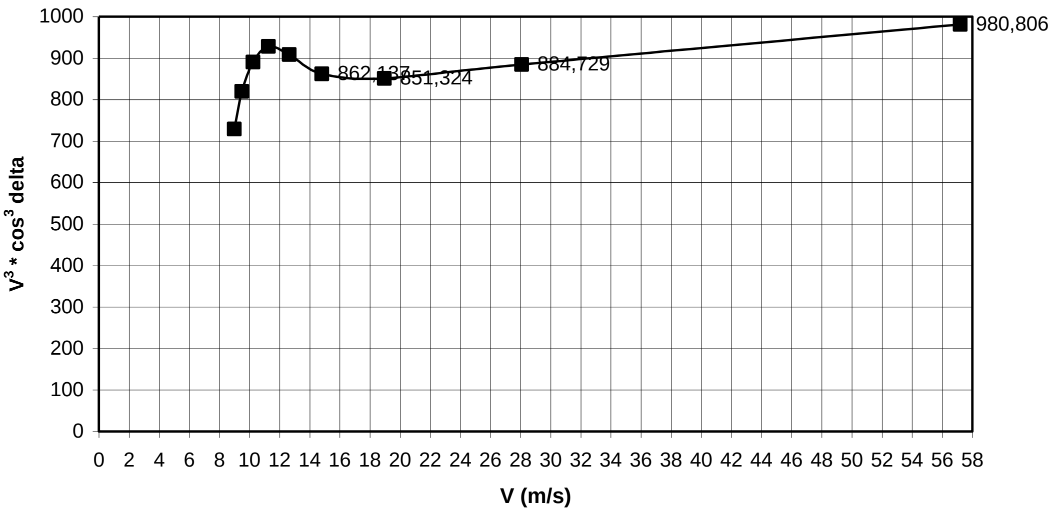

fig. 10 $V^3 * \cos^3\delta$ as a function of V for the pendulum safety system with a torsion spring and a Gö 623 airfoil for V_d = 9 m/s

The maximum power for wind speeds below 40 m/s is generated at a wind speed of 11.26 m/s belonging to a yaw angle δ = 30°. This wind speed is called the rated wind speed V_{rated}. The power at V_{rated} = 11.26 m/s is a factor 928 /729 = 1.273 higher than at V_d = 9 m/s. The power is the mechanical power supplied by the rotor shaft. For the electrical power, the generator efficiency has to be taken into account.

4 Description of the pendulum safety system with a torsion spring and a rotor with a 7.14 % cambered airfoil

The formulas given in chapter 3 are valid for a rotor designed with a constant chord and an airfoil with a flat lower side like the Gö 623, the Gö 624 or the Gö 711. If the airfoil is a 7.14 % cambered sheet, like used for some of the VIRYA windmills with steel blades, some formulas out of chapter 3 have to be changed. The self orientating moment has the double value and the thrust coefficient is somewhat higher (0.75 in stead of 0.7). The maximum power coefficient is somewhat lower (about 0.38). Formulas 9, 10, 11, 12, 13 and 14 change into 30, 31, 32, 33, 34 and 35.

$C_{so} = 0.045 \sin 3\delta$ (-) (for 0° < δ < 40°) (30)

$C_{so} = 0.0664 \cos^2\delta$ (-) (for 40° < δ < 90°) (31)

If the direction of the moment for a negative value of δ is taken the same as for a positive value of δ, formula 9 can also be used for –40° < δ < 0°. The path of both curves is given in figure 11.

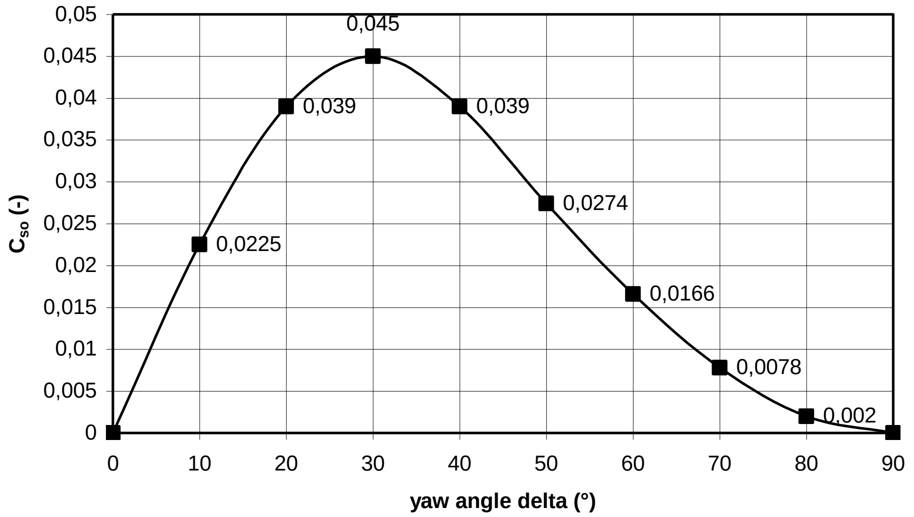

fig. 11 Path of C_{so} as a function of the yaw angle δ

(8) + (30) gives:

$M_{so} = 0.045 \sin 3\delta * ½\rho V^2 * \pi R^3$ (Nm) (for 0° < δ < 40°) (32)

(8) + (31) gives:

$M_{so} = 0.0664 \cos^2\delta * ½\rho V^2 * \pi R^3$ (Nm) (for 40° < δ < 90°) (33)

(2) + (7) + (32) and e = 0.2 R gives:

$M_{rotor} = 0.2 * C_t * \cos^2\delta * ½\rho V^2 * \pi R^3 - 0.045 \sin 3\delta * ½\rho V^2 * \pi R^3$ or
$M_{rotor} = ½\rho V^2 * \pi R^3 (0.2 * C_t * \cos^2\delta - 0.045 \sin 3\delta)$ (Nm)
(for 0° < δ < 40°) (Nm) (34)

(2) + (7) + (33) and e = 0.2 R gives:

$M_{rotor} = 0.2 * C_t * \cos^2\delta * ½\rho V^2 * \pi R^3 - 0.0664 \cos^2\delta * ½\rho V^2 * \pi R^3$ or
$M_{rotor} = ½\rho V^2 * \pi R^3 * \cos^2\delta (0.2 * C_t - 0.0664)$ (Nm)
(for 40° < δ < 90°) (Nm) (35)

(17) + (20) + (21) + (34) gives:

$½\rho V^2 * \pi R^3 * (0.2 * C_t * \cos^2\delta - 0.045 \sin 3\delta) = G * R_G * \{\sin(30° – \delta) + \delta/65°\}$ or
$½\rho V^2 * (0.2 * C_t * \cos^2\delta - 0.045 \sin 3\delta) * \pi R^3 / (G * R_G) = \sin(30° – \delta) + \delta/65°$
(for 0° < δ < 40°) (36)

(17) + (20) + (21) + (35) gives:

$½\rho V^2 * \pi R^3 \cos^2\delta * (0.2 * C_t - 0.0664) = G * R_G * \{\sin(30° – \delta) + \delta/65°\}$ or
$½\rho V^2 * \cos^2\delta * (0.2 * C_t - 0.0664) * \pi R^3 / (G * R_G) = \sin(30° – \delta) + \delta/65°$
(for 40° < δ < 90°) (37)

The design wind speed V_d is defined as the wind speed for which the head just starts moving backwards but for which δ is still just 0°. The design wind speed for a rotor with 7.14 % cambered airfoil is taken lower than for a rotor with a Gö 623 airfoil because the much higher self orientating moment would otherwise result in a too high value for the rated wind speed. It is assumed that all parameters of the safety system are chosen such that $V_d = 7$ m/s. For $V = V_d$ and $\delta = 0°$ formula 36 changes into:

$0.2 * C_t * ½\rho V_d^2 * \pi R^3 / (G * R_G) = \sin 30°$ or

$R^3 / (G * R_G) = \sin 30° / (0.2 * C_t * \pi * ½\rho V_d^2)$ (38)

Formula 38 is the same as formula 24. The thrust coefficient C_t is about 0.75 for a rotor running at the design tip speed ratio and blades with a 7.14 % cambered airfoil. The air density ρ is about 1.2 kg/m^3 for air of 20 °C at sea level. It is chosen that V_d = 7 m/s. Substitution of these values in formula 38 gives:

$R^3 / (G * R_G) = 0.03609$ (for V_d = 7 m/s)   (39)

The rotor and head geometry has to be chosen such that equation 39 is fulfilled.

(36) + (39) gives:

$0.03609 * ½\rho V^2 * (0.2 * C_t * \cos^2\delta - 0.045 \sin 3\delta) * \pi = \sin(30° – \delta) + \delta/65°$
(for $0° < \delta < 40°$)   (40)

Substitution of ρ = 1.2 kg/m^3 and C_t = 0.75 (-) in formula 40 gives:

$0.06803 * V^2 * (0.15 * \cos^2\delta - 0.045 \sin 3\delta) = \sin(30° – \delta) + \delta/65°$ or

$V = \sqrt{} [\{\sin(30° – \delta) + \delta/65°\} / \{0.06803 * (0.15 * \cos^2\delta - 0.045 \sin 3\delta)\}]$
(for $0° < \delta < 40°$)   (41)

(37) + (39) gives:

$0.03609 * ½\rho V^2 * \cos^2\delta * (0.2 * C_t - 0.0664) * \pi = \sin(30° – \delta) + \delta/65°$
(for $40° < \delta < 90°$)   (42)

Substitution of ρ = 1.2 kg/m^3 and C_t = 0.75 (-) in formula 42 gives:

$0.06803 * V^2 * \cos^2\delta * (0.15 - 0.0664) = \sin(30° – \delta) + \delta/65°$ or
$0.00569 * V^2 * \cos^2\delta = \sin(30° – \delta) + \delta/65°$ or

$V = \sqrt{} [\{\sin(30° – \delta) + \delta/65°\} / (0.00569 * \cos^2\delta)]$ (for $40° < \delta < 90°$)   (43)

Next formula 41 and 43 are used to calculate V for different values of δ. It has been chosen that δ = 0°, 10°, 20°, 30°, 40°, 50°, 60°, 70° and 80°. The result is given in table 2.

δ (°)	V (m/s)	cosδ	$\cos^2\delta$	$\cos^3\delta$	V * cosδ	$V^2 * \cos^2\delta$	$V^3 * \cos^3\delta$
0	7	1	1	1	7	49	343
10	7.6987	0.9848	0.9698	0.9551	7.582	57.480	436.001
20	8.6998	0.9397	0.8830	0.8298	8.175	66.831	546.388
30	10.0254	0.8660	0.75	0.6495	8.682	75.381	654.462
40	11.5054	0.7660	0.5868	0.4495	8.813	77.677	684.597
50	13.4802	0.6428	0.4132	0.2656	8.665	75.085	650.605
60	17.2458	0.5	0.25	0.125	8.623	74.354	641.151
70	25.5391	0.3420	0.1170	0.0400	8.734	76.313	666.311
80	52.0441	0.1736	0.0302	0.00524	9.035	81.799	738.662

table 2 Calculated relation in between δ and V for V_d = 8 m/s

In table 1 the values for cosδ, $\cos^2\delta$, $\cos^3\delta$, V * cosδ, $V^2 * \cos^2\delta$ and $V^3 * \cos^3\delta$ are also mentioned. V * cosδ is an indication for the increase of the rotational speed (see formula 1). $V^2 * \cos^2\delta$ is an indication for the increase of the thrust and the torque (see formula 2 and 3). $V^3 * \cos^3\delta$ is an indication for the increase of the power (see formula 4). The values for δ as a function of V are given as the δ-V curve figure 12.

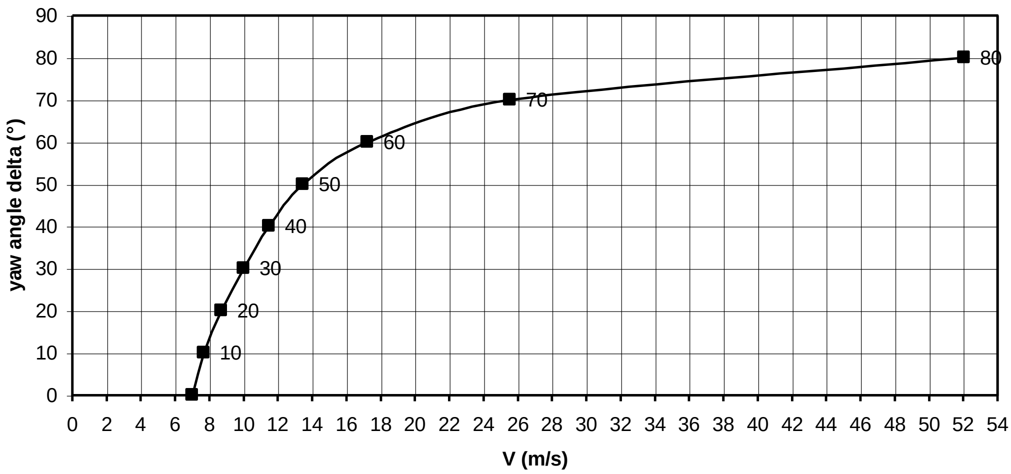

fig. 12 Calculated δ-V curve for the pendulum safety system with a torsion spring and a 7.14 % cambered airfoil for V_d = 7 m/s

In table 2 and figure 12 it can be seen that a very large increase of the wind speed (from 25.5391 m/s up to 52.0441 m/s) is needed to increase the yaw angle from 70° to 80°. The helicopter position (for δ = 90°) will never be reached for stationary conditions. However, hard wind gusts at hurricanes or tornados will cause swinging movements resulting in reaching δ = 90° for a short moment. To make the system tornado proof, it must be possible to fix the movement at δ = 90°, so there must be a stop and a kind of clamp at this angle. This clamp can work automatically and it will lock the rotor at δ = 90°. If this has happened, one has to unlock the system manually but as automatic locking will happen only during very high wind gusts, automatic locking is an acceptable option. It must be possible to turn the rotor to the helicopter position and to unlock it while standing on the tower at about 2/3 of the height.

High upwards wind speeds may occur in the central part of a tornado, so the rotor may start rotating even if the safety system is locked in the helicopter position. However, the rotor will rotate backwards if the wind comes from below and the airfoil will therefore have a lot of drag which strongly limits the rotational speed and the thrust. So the rotor will survive large upwards wind speeds. But there may be extremely strong tornados which will not be survived.

If figure 12 is compared to figure 7 it can be seen that figure 12 is flatter which is caused by the much larger self orientating moment.

The calculated values of V * cosδ as a function of V are given in figure 13. This curve is an indication of the variation of the rotational speed for $V > V_d$.

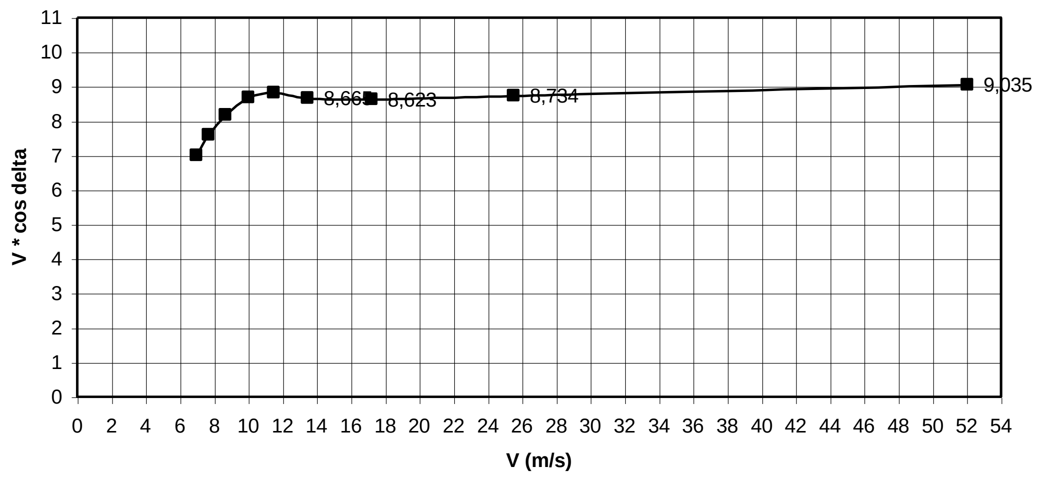

fig. 13 V * cosδ as a function of V for the pendulum safety system with a torsion spring and a 7.14 % cambered airfoil for $V_d = 7$ m/s

In figure 13 it can be seen that the rotational speed is sharply limited and almost constant for 10 < V < 54 m/s. The maximum rotational speed at moderate wind speeds is reached at a yaw angle of 40° belonging to a wind speed of rounded 11.5 m/s. This demonstrates the reason why the design wind speed for a rotor with a 7.14 % cambered airfoil is taken 7 m/s in stead of 9 m/s for a rotor with a Gö 623 airfoil.

The calculated values of $V^2 * \cos^2\delta$ as a function of V are given in figure 14. This curve is an indication of the variation of the thrust and the torque for $V > V_d$.

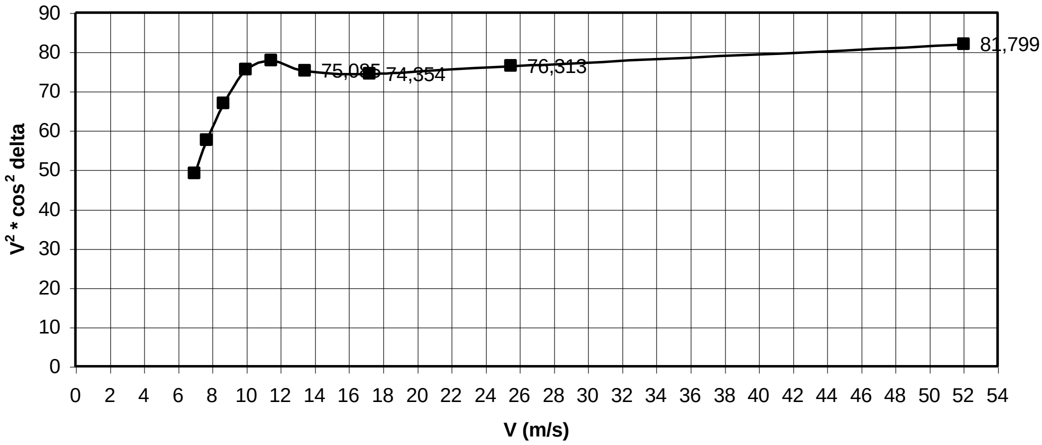

fig. 14 $V^2 * \cos^2\delta$ as a function of V for the pendulum safety system with a torsion spring and a 7.14 % cambered airfoil for V_d = 7 m/s

In figure 14 it can be seen that the thrust and the torque have a peak at V = 11.5 m/s but also that they are almost constant for 10 < V < 54 m/s.

The calculated values of $V^3 * \cos^3\delta$ as a function of V are given in figure 15. This curve is an indication of the variation of the power for $V > V_d$.

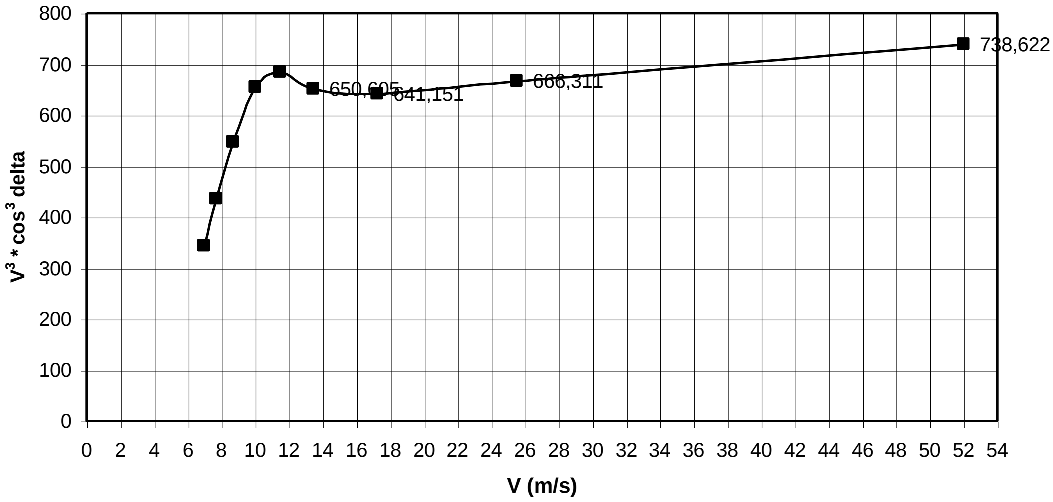

fig. 15 $V^3 * \cos^3\delta$ as a function of V for the pendulum safety system with a torsion spring and a 7.14 % cambered airfoil for V_d = 7 m/s

The maximum power for wind speeds below 34 m/s is generated at a wind speed of rounded 11.5 m/s belonging to a yaw angle δ = 40°. This wind speed is called the rated wind speed V_{rated}. The power at V_{rated} = 11.5 m/s is a factor 684.6 / 343 = 1.996 higher than at V_d = 7 m/s.

This ratio is higher than the ratio of 1.273 which is valid for a rotor with a Gö airfoil. This is caused by the much higher self orientating moment.

The power is the mechanical power supplied by the rotor shaft. For the electrical power, the generator efficiency has to be taken into account.

5 Following variations of the wind direction

The horizontal hinge axis must be kept perpendicular to the wind direction, so the head must turn into the wind. This yawing of the head around the tower axis causes a gyroscopic moment perpendicular to the plane of the rotor axis and the tower axis. So this gyroscopic moment has a tendency to topple the rotor along the horizontal hinge axis. The direction of the gyroscopic moment depends on the direction of rotation of the rotor shaft and of the direction of rotation of the head along the tower axis. The rotor turns always in the same direction but the yawing along the tower axis is half the time left hand and half the right hand. So the gyroscopic moment has half the time a tendency to increase δ and half the time a tendency to decrease δ. As the safety system should mainly be influenced by variations of wind speed and preferably not by variations of wind direction, turning of the head into the wind must be done slowly.

A normal vane will turn the head too fast at high wind speeds. A system with side rotors which drive the head in the wind through a reducing gearing gives a very low yawing speed around the tower axis but this system is rather complicated and expensive. I have tested a so called double vane system on one of my earliest windmills, the DRIEKA-4, which had a rotor diameter of 4 m and a (very noisy) safety system with elastic air brakes on the blade tips.

It was a long horizontal pipe parallel to the rotor with a square sheet on each end of the pipe. Each sheet makes an angle of 20° with the rotor axis and the direction of the angles is chosen such that touching lines along the sheets intersect with the rotor axis before the rotor. Each sheet had a width and height of 500 mm and was made of 4 mm steel sheet. A photo of the DRIEKA-4 is given in figure 16.

fig. 16 DRIEKA-4 windmill with double vane and tree tower designed in about 1985

The moment of inertia of this vane is very large and fast head movements are therefore damped very well. For a windmill with the pendulum safety system, the optimum position of the pipe might be that its axis coincides with the horizontal hinge axis. For this position, the pipe is not hindering the swinging movement of the head. Because the hinge axis intersects with the tower axis, the weight of the vane exerts no moment on the yaw bearing housing.

6 Ideas about the torsion spring

Formula 20 gives the relation which has to be fulfilled to get a torsion spring which has the characteristic as given in figure 5. This formula shows that $M_{torsion} = 0$ for $\delta = 0°$ and that $M_{torsion}$ is $G * R_G$ for $\delta = 65°$. G and R_G have to be chosen such that formula 25 or formula 39 are fulfilled for the chosen value of R, to realise the correct design wind speed. G is mainly the result of the weight of the rotor and the generator. It might be that the product of $G * R_G$ is too large for the chosen rotor and generator. In this case two balancing weights have to be added at the backside of the head to realise the correct product of $G * R_G$. The head must be designed such that the balancing weights don't hit the tower pipe. The head geometry has to be chosen such that $\alpha_0 = 30°$ (see figure 3).

If the vane system is chosen which is described in chapter 4, it is logic to use two torsion springs which are mounted in the vane pipes. Each spring must give half the required torsion moment. So the torsion moment of one spring $M_{torsion\ 1sp}$ is given by:

$$M_{torsion\ 1sp} = G * R_G * \delta/130° \qquad (Nm) \qquad (44)$$

A torsion spring can be made out of a spring steel strip of which one end is connected to the end of a pipe and the other part is connected to the bearing housing of the horizontal hinge axis. The torsion stiffness of the pipe is very large with respect to the torsion stiffness of the strip, so it can be assumed that only the strip is twisted. The specific torsion angle ϕ (in rad/mm) for a strip with a height h and a width b and a length of 1 mm is given by:

$$\phi = 3.6\ M / \{t_m * b^3 * h^3 / (b^2 + h^2)\} \qquad (rad/mm) \qquad (45)$$

M is the torsion moment (in Nmm). t_m is the torsion modulus which is about $8.15 * 10^4$ N/mm^2 for spring steel. b and h are given in mm and h > b.

The maximum torsion stress τ is given by:

$$\tau = M / (2/9\ b^2 * h) \qquad (N/mm^2) \qquad (46)$$

The determination of the spring geometry is out of the scope of this report because this can't be done without making a composite drawing of the head.

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