Development of an ecliptic safety system with a torsion spring

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February 2009
modified April 2012 (chapter 8 added)

KD 409

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1 Introduction

Windmills with fixed rotors can be protected against too high forces and too high rotational speeds by turning the rotor out of the wind. This can be done around a vertical and around an horizontal axis. All present VIRYA windmills developed by Kragten Design turn out of the wind around a vertical axis and make use of the so called hinged side vane safety system.

Safety systems for water pumping windmills are described in report R 999 D (ref. 1). Water pumping windmills normally have fixed rotors and all safety system are working by turning the rotor out of the wind. However, electricity generating windmills can also be protected by turning the rotor out of the wind. In chapter 2 of R 999 D, the reasons are given why a safety system is necessary. These reasons are:

1 Limitation of the axial force or thrust on the rotor to limit the load on the rotor blades, the tower and the foundation.
2 Limitation of the rotational speed of the rotor to limit the centrifugal force in the blades, imbalance forces, high gyroscopic moments in the blades and the rotor shaft, to prevent flutter for blades with low torsion stiffness and to prevent too high rotational speeds of the load which is relevant for limitation of heat dissipation in a generator or for limitation of shock forces in the transmission to a piston pump.
3 Limitation of the yawing speed to limit high gyroscopic moments in the blades and the rotor shaft.

Almost all known systems are described shortly in chapters 3 and 4 of R 999 D. The three most generally used systems, the ecliptic system, the inclined hinge main vane system and the hinged side vane system are described in detail in chapter 7 of report R 999 D. Because report R 999 D is no longer available, the hinged side vane system is also described in several KD-reports. This system is described for the VIRYA-4.2 windmill in report KD 213 (ref. 2).

Every safety system has certain advantages and disadvantages. The main advantages of the hinged side vane safety system are:

1) It is simple and cheap.
2) It has a δ-V curve which is lying close to the ideal δ-V curve (see chapter 2).
3) The hinge axis is loaded only lightly and therefore simple door hinges can be used.
4) The vane blade is situated in the undisturbed wind and therefore a relatively small vane blade area is required to generate a certain aerodynamic force.
5) The moment of inertia of the head is large resulting in low yawing speeds and so large gyroscopic moments at high wind speeds are prevented.

The main disadvantages of the hinged side vane system are:

1) There must be a certain ratio in between the vane area and the vane weight if a certain rated wind speed is wanted. Therefore it appears to be difficult to make a large vane blade stiff enough. The hinged safety system is therefore limited to windmills with a maximum rotor diameter of about 5 m.
2) The system is sensible to flutter of the vane blade, if the vane blade and the vane arm is not made stiff enough. Flutter is suppressed effectively using a vane blade stop at the almost horizontal position of the vane blade
3) It is difficult to turn the head out of the wind permanently by placing the vane blade in the horizontal position because this vane blade is positioned far form the tower and far from the ground.

In report KD 377 (ref. 3) a safety system is described with which the rotor is turned out of the wind around an horizontal axis. This system has rather good characteristics which can easily be determined. However, this system needs an extra horizontal axis and it requires rather heavy balancing weights. It will be investigated if it is possible to realise about similar characteristics for an ecliptic system equipped with a torsion spring. In report KD 408 (ref. 4) an ecliptic safety system is described for which the eccentricity is chosen so large that the contribution of M_{so} and $F_{s\delta}$ to M_{rotor} can be neglected. This results in a simple equation for M_{rotor}. However, a large eccentricity results in a very large vane blade which is impractical.

2 The ideal δ-V curve

Generally it is wanted that the windmill rotor is perpendicular to the wind up to the rated wind speed V_{rated}, and that the rotor turns out of the wind such that the rotational speed, the rotor thrust, the torque and the power stay constant above V_{rated}. It appears to be that the component of the wind speed perpendicular to the rotor plane determines these four quantities. The yaw angle δ is the angle in between the wind direction and the rotor axis. The component of the wind speed perpendicular to the rotor plane is therefore $V \cos\delta$. The formulas for a yawing rotor for the rotational speed n_δ, the rotor thrust $F_{t\delta}$, the torque Q_δ and the power P_δ are given in chapter 7 of report KD 35 (ref. 5). These formulas are copied as formula 1, 2, 3 and 4.

$$n_\delta = 30 * \lambda * \cos\delta * V / \pi R \qquad \text{(rpm)} \tag{1}$$

$$F_{t\delta} = C_t * \cos^2\delta * \tfrac{1}{2}\rho V^2 * \pi R^2 \qquad \text{(N)} \tag{2}$$

$$Q_\delta = C_q * \cos^2\delta * \tfrac{1}{2}\rho V^2 * \pi R^3 \qquad \text{(Nm)} \tag{3}$$

$$P_\delta = C_p * \cos^3\delta * \tfrac{1}{2}\rho V^3 * \pi R^2 \qquad \text{(W)} \tag{4}$$

These four quantities stay constant above V_{rated} if the component of the wind speed perpendicular to the rotor plane is kept constant above V_{rated}. So in formula:

$$V \cos\delta = V_{rated} \qquad (\text{for } V > V_{rated}) \tag{5}$$

It is assumed that the rotor is loaded such that it runs at the design tip speed ratio λ_d. If the wind speed is in between 0 m/s and V_{rated}, the n-V curve is a straight line through the origin. The F_t-V and the Q-V curves are then parabolic lines and the P-n curve is a cubic line.

Formula 5 can be written as:

$$\delta = \arccos\,(V_{rated} / V) \qquad (°) \tag{6}$$

This formula is given as a graph in figure 1 for different value of V / V_{rated}. The value of δ has been calculated for V / V_{rated} is respectively 1, 1.01, 1.05, 1.1, 1.25, 1.5, 2, 2.5, 3, 4, 5 and 6.

The rated wind speed V_{rated} is chosen on the basis of the maximum thrust and the maximum rotational speed which is allowed for a certain rotor and a certain generator. Mostly V_{rated} is chosen about 10 m/s. For the chosen value of V_{rated}, figure 1 can be transformed into the δ-V curve for which V (in m/s) is given on the x-axis. If it is chosen that V_{rated} = 10 m/s, figure 1 becomes the δ-V curve if all values on the x-axis are multiplied by a factor 10.

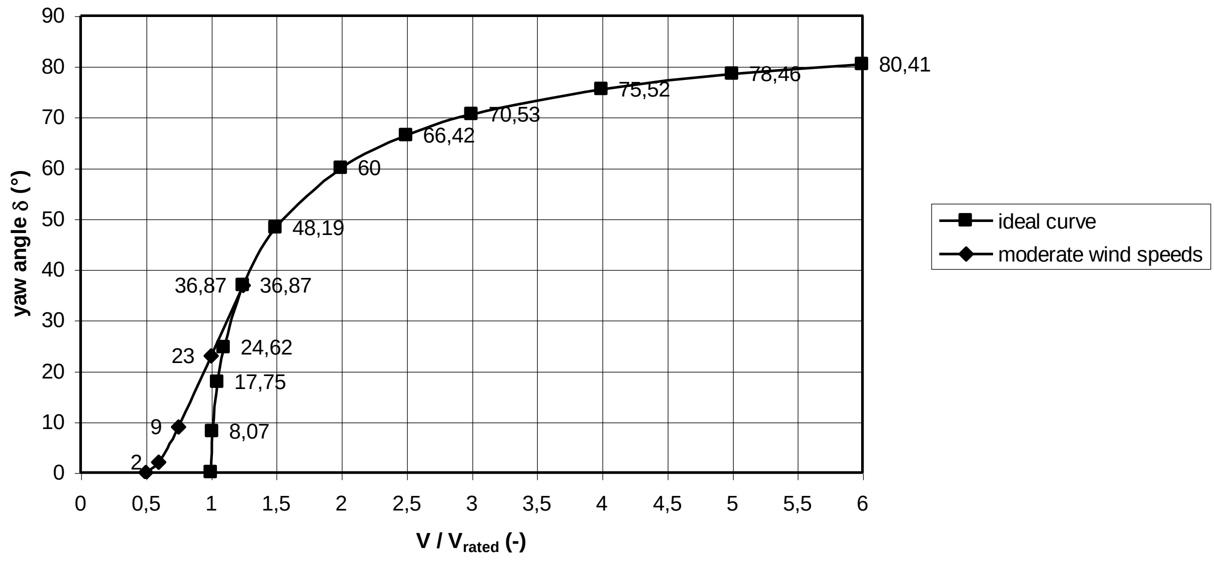

fig. 1 the δ-V/V_{rated} curve for the ideal safety system

In figure 1 it can be seen that the rotor is perpendicular to the wind for (V / V_{rated}) < 1 but that the required change in δ is very sudden if V / V_{rated} is a very little higher than 1. So even if one would have a safety system which theoretically has the ideal δ-V curve, in practice this curve will not be followed because the inertia of the system prevents sudden changes of δ around V / V_{rated} = 1. So the system will turn out of the wind less than according to the ideal δ-V curve. This will result in a certain overshoot of the rotational speed and the thrust.

For high values of V / V_{rated}, a certain increase of V, and therefore a certain increase of V / V_{rated}, requires only a relatively small increase of δ. It is therefore much easier to follow the theoretical δ-V curve at high wind speeds.

Because of this effect, it is practically impossible to follow the ideal δ-V curve for wind speeds lower than about 1.25 * V_{rated} and the practical δ-V curve must therefore start increasing already at a much lower wind speed than the theoretical rated wind speed. An example of a practical δ-V curve for moderate wind speeds is also given in figure 1. Even for this practical curve for moderate wind speeds and for the ideal curve for values of V / V_{rated} > 1.25, there will be a certain overshoot of n and F_t because of inertia effects.

3 Description of the normal ecliptic safety system

The ecliptic safety system is widely used in old fashioned water pumping windmills. The name of the system probably comes from the windmill of manufacture Eclipse which is equipped with this system. The ecliptic system is described in general in chapter 4.4 and in detail in chapter 7.4 of report R999D (ref. 1). The ecliptic system can be used in combination with an eccentrically placed rotor or with a centrally placed rotor and an auxiliary vane. Only the use in combination with an eccentrically placed rotor will be taken into account.

The main feature of a normal ecliptic safety system is that the main vane is turning around a vertical axis and that the vane arm it is pulled against a stop by a tension spring. The geometry of the rotor, the head and the vane are chosen such that the rotor is perpendicular to the wind direction as long as the vane arm makes contact with the stop. The pulling force in the spring exerts a certain moment M_{spring} around the vane axis. M_{spring} depends on the distance in between the hart of the spring and the vane axis and so on the position of the vane arm. The pulling force in the spring depends on the spring characteristics.

The vane arm is touching the stop as long as M_{spring} is larger than the moment M_{vane} exerted by the aerodynamic force on the vane blade around the vane axis. At a certain critical wind speed called V_{crit}, M_{vane} becomes larger than M_{spring} and the vane turns away.

The rotor exerts a certain rotor moment M_{rotor} around the tower axis. This rotor moment is mainly determined by the rotor thrust $F_{t\delta}$ and the eccentricity e but the side force on the rotor $F_{s\delta}$ in combination with the distance f in between the rotor plane and the tower axis and the so called self orientating moment M_{so} also have a certain influence. If M_{rotor} becomes larger than the moment of the vane around the tower axis, the rotor starts turning out of the wind. This is the case for wind speeds higher than V_{crit} when the vane arm is no longer in contact with the stop.

How the rotor turns out of the wind as a function of the wind speed is difficult to determine, especially if the vane blade is in the rotor shadow where the wind speed is not well known. Another disadvantage of the ecliptic system is that, if the rotor is turned out of the wind a lot and if the wind speed suddenly decreases, the vane arm will move back to its zero position. It will hit the stop with a large force if the stop is not elastic. To prevent that the vane arm can touch the rotor at high wind speeds, another stop is needed at a position where the vane arm is about parallel to the rotor plane.

4 Description of the ecliptic system with torsion spring

The new ecliptic system will have some special differences if compared to the normal system. The differences are:

a) A torsion spring will be used. The spring moment M_{spring} will therefore increase linear to the angle γ over which the vane arm turns.

b) The vane arm will point upwards with an angle of 45° and will be so long that the vane blade is in the undisturbed wind speed V. The vane blade will be square and will be positioned such that two sides are horizontal. The angle in between the vane blade and the wind direction is called α.

c) There will be an elastic stop at the zero position of the vane arm for $\gamma = 0°$ and a second elastic stop at a position for $\gamma = 100°$. So the shock forces are limited if the vane arm hits one of these stops and the vane arm can never touch the rotor. However, the elasticity of the stop at zero position is neglected for the determination of γ, so it is assumed that $\gamma = 0°$ as long as the vane arm touches this stop.

d) The position of the zero line of the vane arm is chosen such that there is an angle $\varepsilon = 20°$ in between this zero line and the rotor axis.

e) The geometry of rotor, head and vane are chosen such that the rotor axis for a rotating rotor is perpendicular to the wind for $\gamma = 0°$. The left hand angle in between the rotor axis and the wind direction is called δ. So $\delta = 0°$ and $\alpha = \varepsilon = 20°$ for this condition.

f) The torsion moment at $\gamma = 0°$ is called $M_{spring0}$. The spring constant of the torsion spring is chosen such that the torsion moment for $\gamma = 100°$, $M_{spring100}$, is twice the value as $M_{spring0}$. This ratio is chosen because the moment M_G for the pendulum safety system, produced by the balancing weights at V_{rated}, is also twice the value of M_G at V_d.

g) The eccentricity e in between the rotor axis and the tower axis will be taken rather large with respect to the rotor radius R (e = 0.2 R). The contribution of the side force $F_{s\delta}$ and the self orientating moment M_{so} to the rotor moment M_{rotor} will therefore be rather small. However, they can't be neglected for this ratio in between e and R.

h) The position of the vane axis is chosen such that it coincides with the tower axis. This has as advantage that the vane moment around the vane axis is the same as around the tower axis and this simplifies the moment equations.

In point e it is said that the rotor axis is perpendicular to the wind for $\gamma = 0°$. However, this is only true for a rotating rotor which turns about with the design tip speed ratio. The thrust coefficient for a non rotating rotor is much lower than for a rotating rotor which means that the thrust force and so also M_{rotor}, will be much lower too. This means that the rotor axis will have a negative yaw angle δ when the rotor is not rotating at low wind speeds.

The wind speed for which M_{rotor} is the same as M_{spring} for $\gamma = 0°$, is called the design wind speed V_d. In analogy to the pendulum safety system, it is chosen that $V_d = 7$ m/s. The rotor will be perpendicular to the wind for wind speeds lower than V_d (as long as the rotor is rotating at a about the design tip speed ratio). This situation for $V = V_d = 7$ m/s is given in figure 2 for a top view of the head.

The wind speed for which the rotational speed, the thrust and the power have a maximum, is called the rated wind speed V_{rated}. V_{rated} is determined in chapter 6 and it appears that V_{rated} is rather high for the chosen characteristics of the torsion spring. For very high wind speeds, the angle α in between the wind direction and the vane blade will be very small and the yaw angle δ will therefore be almost 80° for $\gamma = 100°$.

The rotor will turn out of the wind for wind speeds higher than V_d. The yaw angle δ depends on the undisturbed wind speed V. The situation for a wind speed V = 10.775 m/s (see table 3) is given in figure 3 for a top view of the head. In table 3 it can be seen that $\delta = 30°$, $\alpha = 8.44°$ and $\gamma = 41.56°$ for V = 10.775 m/s.

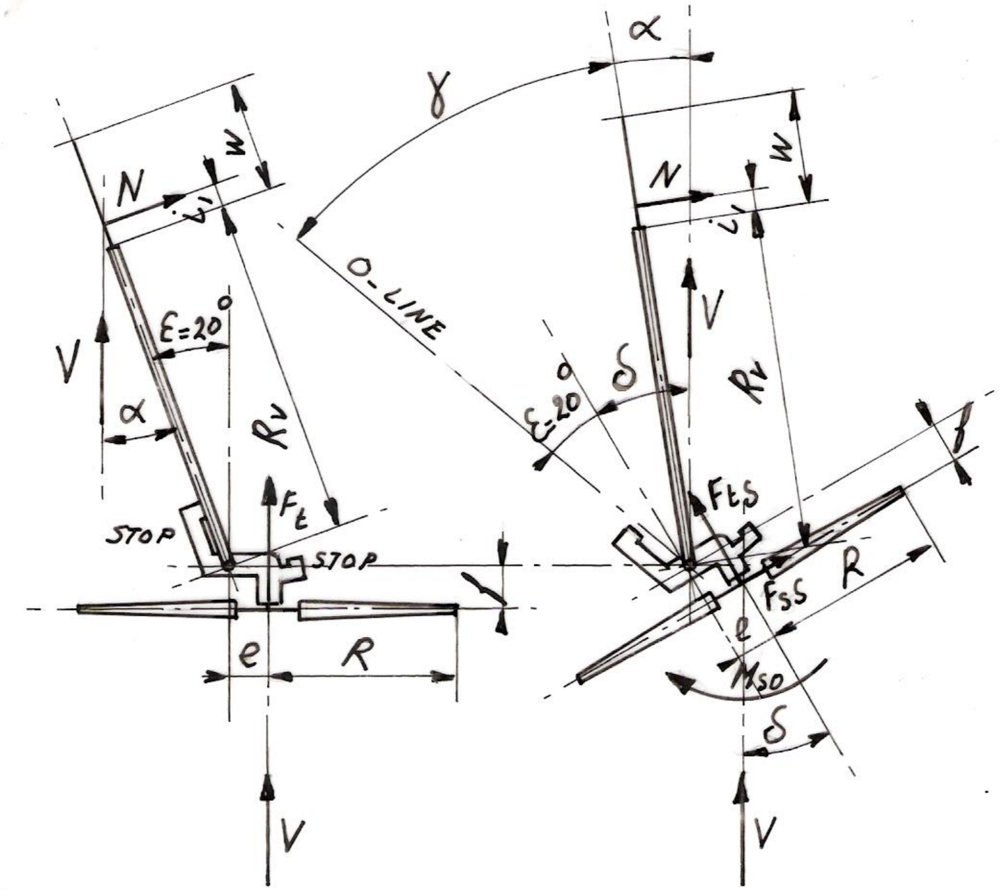

fig. 2 Situation for $V = V_d = 7$ m/s

fig. 3 Situation for V = 10.775 m/s

In figure 3 it can be seen that:

$$\delta = \gamma + \alpha - \varepsilon \quad (°) \tag{7}$$

5 The moment equations

The moment produced by the aerodynamic normal force N which is working on the vane blade is called M_{vane}. Now it is assumed that the system is functioning quasi-stationary. So no moments are needed for acceleration of the system from one position into the other. This means that:

$$M_{rotor} = M_{vane} \qquad (8)$$

For V > V_d, the vane arm is no longer touching the stop at zero position. The force at the stop is also zero for V = V_d. So in both cases it is valid that:

$$M_{rotor} = M_{spring} \quad (\text{for } V \geq V_d) \qquad (9)$$

For V < V_d, a part of M_{spring} is taken by the stop and another part is taken by the vane. Which part depends on the elasticity of the stop and this results in a small variation of γ, but this aspect is not taken into account.

For the determination of the δ-V curve, it is necessary to determine the formulas for M_{rotor}, M_{spring} and M_{vane}.

5.1 Determination of M_{rotor}

M_{rotor} is caused by the influence of the thrust force $F_{t\delta}$, the side force $F_{s\delta}$ and the so called self orientating moment M_{so}. $F_{t\delta}$ results in a moment $M_{Ft\delta}$. $F_{s\delta}$ results in a moment $M_{Fs\delta}$. Both $M_{Ft\delta}$ and $M_{Fs\delta}$ have a right hand direction which tends to increase δ. However, M_{so} has a left hand direction such that δ is decreased. M_{rotor} is therefore given by:

$$M_{rotor} = M_{Ft\delta} + M_{Fs\delta} - M_{so} \quad (\text{Nm}) \qquad (10)$$

$$M_{Ft\delta} = F_{t\delta} * e \quad (\text{Nm}) \qquad (11)$$

(2) + (11) gives:

$$M_{Ft\delta} = C_t * \cos^2\delta * ½\rho V^2 * \pi R^2 * e \quad (\text{Nm}) \qquad (12)$$

$$M_{Fs\delta} = F_{s\delta} * f \quad (\text{Nm}) \qquad (13)$$

The distance in between the rotor plane and the tower centre is called f. For the side force on the rotor $F_{s\delta}$, no formula is given in report KD 35 (ref. 5). If one would calculate $F_{s\delta}$ with the component of the wind speed in the rotor plane, V sinδ, the side force would be proportional to $\sin^2\delta$. However, from measurements (see figure 23, report R 999 D) it is found that $F_{s\delta}$ increases much faster than a $\sin^2\delta$ function for small values of δ. A sinδ function gives a better approximation.

For very large angles δ, the tip speed of the rotor is only little with respect to the wind speed. The side area of the rotor A_s, then can be seen as a drag area with a drag coefficient C_d. The ratio i in be between A_s and the swept rotor area $\pi * R^2$ depends on the type of rotor. For fast running rotors as used in the VIRYA windmills, A_s is very small with respect to the swept rotor area because the chord, the airfoil thickness and the blade angles are small. In report KD 213 (ref. 2), the two bladed VIRYA-4.2 rotor is taken which has a design tip speed ratio of 8. For this rotor it is determined that i = A_s / ($\pi * R^2$) = 0.01. The drag coefficient C_d depends on the airfoil and is rather low if an aerodynamic airfoil is used.

The yaw angel δ is large at very high wind speeds and the lower blade sees a much larger relative wind speed than the upper blade. It is assumed that the average C_d value for the whole rotor is 1. The side force $F_{s\delta}$ for a yawing rotor is now given by:

$F_{s\delta} = C_d * \sin\delta * ½\rho V^2 * i * \pi R^2$ (N) (14)

(13) + (14) gives:

$M_{Fs\delta} = C_d * f * i * \sin\delta * ½\rho V^2 * \pi R^2$ (Nm) (15)

In KD 35 (ref. 5) no formula is given for the self orientating moment M_{so}. M_{so} is created because the exertion point of the thrust doesn't coincide with the hart of the rotor. There is only little known about M_{so} and only some very rough measurements have been performed which are given in report R 344 D (ref. 6, in Dutch). For these measurement an unloaded two bladed rotor was used with a design tip speed ratio of 5 and provided with a curved sheet airfoil. Practical experience with the VIRYA windmills using a Gö 623 airfoil indicate that M_{so} is much lower for this airfoil. Recently I have made a model of a two bladed rotor with a diameter of 0.8 m with a design tip speed ratio of about 6.5 and using a Gö 623 airfoil. The maximum eccentricity which was possible for which the rotor doesn't turn out of the wind completely, was about 0.027 m. From this measurement it is derived that the maximum self orientating moment for a certain wind speed is about half the value as for the same diameter rotor with a curved sheet airfoil.

M_{so} is given by:

$M_{so} = C_{so} * ½\rho V^2 * \pi R^3$ (Nm) (16)

C_{so} depends on the yaw angle δ and appears to have a maximum for $\delta = 30°$. The estimated C_{so}-δ curve for a rotor with a Gö 623 or similar airfoil can be approximated by two goniometric functions, one function for $0° \leq \delta \leq 40°$ and one function for $40° \leq \delta \leq 90°$. These functions are:

$C_{so} = 0.0225 \sin 3\delta$ (-) (for $0° \leq \delta \leq 40°$) (17)

$C_{so} = 0.0332 \cos^2\delta$ (-) (for $40° \leq \delta \leq 90°$) (18)

If the direction of the moment for a negative value of δ is taken the same as for a positive value of δ, formula 17 can also be used for $-40° \leq \delta \leq 0°$. The path of both curves is given in figure 4.

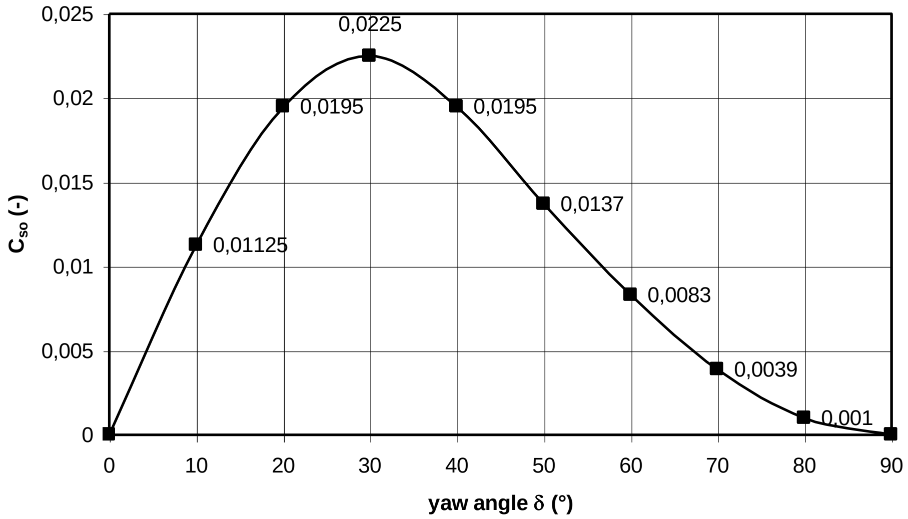

figure 4 Path of C_{so} as a function of the yaw angle δ

(16) + (17) gives:

$M_{so} = 0.0225 \sin 3\delta * ½\rho V^2 * \pi R^3$ (Nm) (for $0° \le \delta \le 40°$) (19)

(16) + (18) gives:

$M_{so} = 0.0332 \cos^2\delta * ½\rho V^2 * \pi R^3$ (Nm) (for $40° \le \delta \le 90°$) (20)

(10) + (12) + (15) + (19) gives:

$M_{rotor} = C_t * e * \cos^2\delta * ½\rho V^2 * \pi R^2 + C_d * f * i * \sin\delta * ½\rho V^2 * \pi R^2 - 0.0225 \sin 3\delta * ½\rho V^2 * \pi R^3$ or

$M_{rotor} = ½\rho V^2 * \pi R^3 (C_t * e/R * \cos^2\delta + C_d * f/R * i * \sin\delta - 0.0225 * \sin 3\delta)$ (Nm) (for $0° \le \delta \le 40°$) (21)

(10) + (12) + (15) + (20) gives:

$M_{rotor} = C_t * e * \cos^2\delta * ½\rho V^2 * \pi R^2 + C_d * f * i * \sin\delta * ½\rho V^2 * \pi R^2 - 0.0332 \cos^2\delta * ½\rho V^2 * \pi R^3$ or

$M_{rotor} = ½\rho V^2 * \pi R^3 (C_t * e/R * \cos^2\delta + C_d * f/R * i * \sin\delta - 0.0332 * \cos^2\delta)$ (Nm) (for $40° \le \delta \le 90°$) (22)

To get an impression of the contribution of $M_{Ft\delta}$, $M_{Fs\delta}$ and M_{so} to M_{rotor}, the moments are made dimensionless by dividing by $½\rho V^2 * \pi R^3$. The formulas 10, 12, 15, 19 and 20 for M_{rotor}. $M_{Ft\delta}$, $M_{Fs\delta}$ and M_{so} change into 23, 24, 25, 26 and 27 for C_{Mrotor}, $C_{MFt\delta}$, $C_{MFs\delta}$, and C_{Mso}.

$C_{Mrotor} = C_{MFt\delta} + C_{MFs\delta} - C_{Mso}$ (-) (23)

$C_{MFt\delta} = C_t * e/R * \cos^2\delta$ (-) (24)

$C_{MFs\delta} = C_d * f/R * i * \sin\delta$ (-) (25)

$C_{Mso} = 0.0225 \sin 3\delta$ (-) (for $0° \le \delta \le 40°$) (26)

$C_{Mso} = 0.0332 \cos^2\delta$ (-) (for $40° \le \delta \le 90°$) (27)

Now the path of $C_{MFt\delta}$, $C_{MFs\delta}$, C_{Mso} and C_{Mrotor} is determined as a function of δ for the VIRYA-4.2 rotor and it is assumed that this rotor is now combined with the ecliptic safety system with a torsion spring. For this rotor it is valid that R = 2.1 m. It is assumed that e = 0.42 m and that f = 0.48 m. So e/R = 0.2 and f/R = 0.2286. It was assumed earlier that i = 0.01. The theoretical thrust coefficient 8/9 = 0.89 for $\lambda = \lambda_d$. However in practice it is a lot lower because the inner part of the rotor is not effective and because a part of the thrust is lost by tip and root losses. Assume $C_t = 0.7$. For the drag coefficient it was earlier assumed that $C_d = 1$. Substitution of these values in formula 24 and 25 gives:

$C_{MFt\delta} = 0.14 \cos^2\delta$ (-) (28)

$C_{MFs\delta} = 0.00229 \sin\delta$ (-) (29)

The moment coefficients are calculated for values of δ in between δ = -40° and δ = 90° rising with 10°. The results of the calculations are given in table 1 and figure 5. If the direction of moments for negative values of δ is taken the same as for positive values of δ, formulas 26, 28 and 29 can also be used for negative values of δ.

δ (°)	$C_{MFt\delta}$ (-)	$C_{MFs\delta}$ (-)	C_{Mso} (-)	C_{Mrotor} (-)
-40	0.08216	-0.00147	-0.01949	0.10018
-30	0.10500	-0.00115	-0.02250	0.12635
-20	0.12362	-0.00078	-0.01949	0.14233
-10	0.13578	-0.00040	-0.01125	0.14663
0	0.14	0	0	0.14
10	0.13578	0.00040	0.01125	0.12493
20	0.12362	0.00078	0.01949	0.10491
30	0.10500	0.00115	0.02250	0.08365
40	0.08216	0.00147	0.01949	0.06414
50	0.05784	0.00175	0.01372	0.04587
60	0.03500	0.00198	0.00830	0.02868
70	0.01638	0.00215	0.00388	0.01465
80	0.00422	0.00226	0.00100	0.00548
90	0	0.00229	0	0.00229

table 1 Calculated values for $C_{MFt\delta}$, $C_{MFs\delta}$, C_{Mso} and C_{Mrotor} for the VIRYA-4.2 rotor

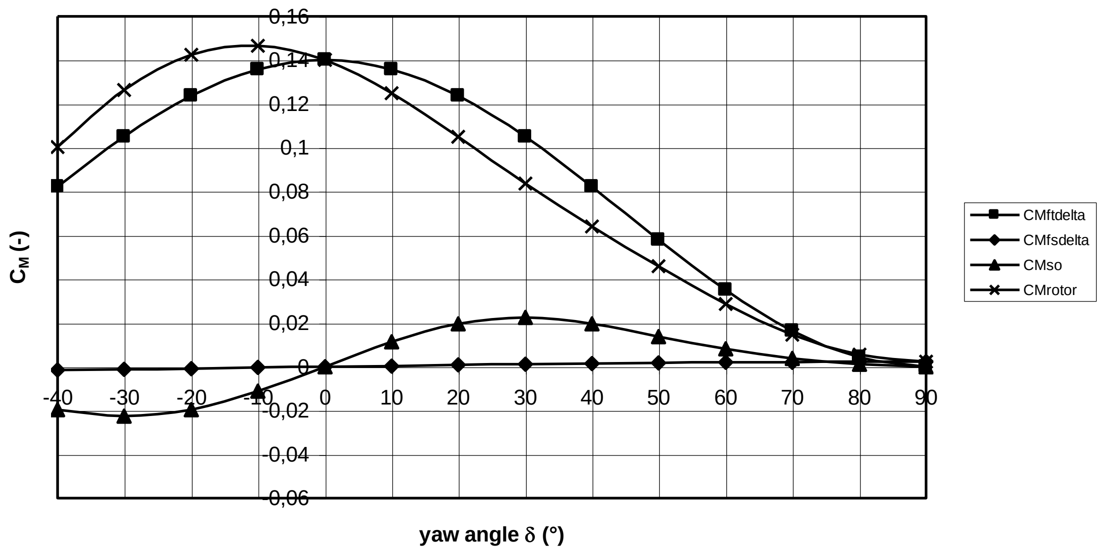

figure 5 Path of $C_{MFt\delta}$, $C_{MFs\delta}$, C_{Mso} and C_{Mrotor} for the VIRYA-4.2 rotor and head geometry

In figure 5 it can be seen that the contribution of $C_{MFs\delta}$ to C_{Mrotor} can be neglected except for very large angles δ. The contribution of C_{Mso} to C_{Mrotor} can not be neglected and causes that the decrease of the C_{Mrotor}-δ curve at increasing δ is much faster than for the $C_{MFt\delta}$-δ curve. For angles δ in between 25° and 60°, C_{Mrotor} is about a factor 0.8 van $C_{MFt\delta}$. C_{Mrotor} has a maximum at about δ = -13°.

The path found in figure 5 for the dimensionless moment coefficients is also valid for the real moments for a certain wind speed.

5.2 Determination of M_{vane}

$$M_{vane} = N * (R_v + i_1) \quad \text{(Nm)} \qquad (30)$$

N is the aerodynamic normal force on the vane blade. R_v is the length of the vane arm in the horizontal plane. i_1 is the distance in between N and the leading edge of the vane blade. This distance is called i_1 in analogy to the name which is used in report KD 213 (ref. 2). i_1 depends on the vane blade shape, on the vane blade width w and on the angle of attack α. The square plate has originally been measured by Flachsbart and the measuring points of these measurements are used to determine the ratio i_1 / w and the C_n-α curve. The ratio i_1 / w for a square vane blade is given as a function of α in figure 7 of report KD 213 (ref. 2). This figure is copied as figure 6.

For a constant wind direction, α will vary in between 20° for wind speeds below V_d (and a rotating rotor) and almost 0° for very high wind speeds. So the average value will be about 10°. This corresponds to a ratio i_1 / w = 0.27. As i_1 is only small with respect to R_v, it is acceptable to use this constant value i_1 / w = 0.27. As the vane blade is square, the height h will be the same as the width w.

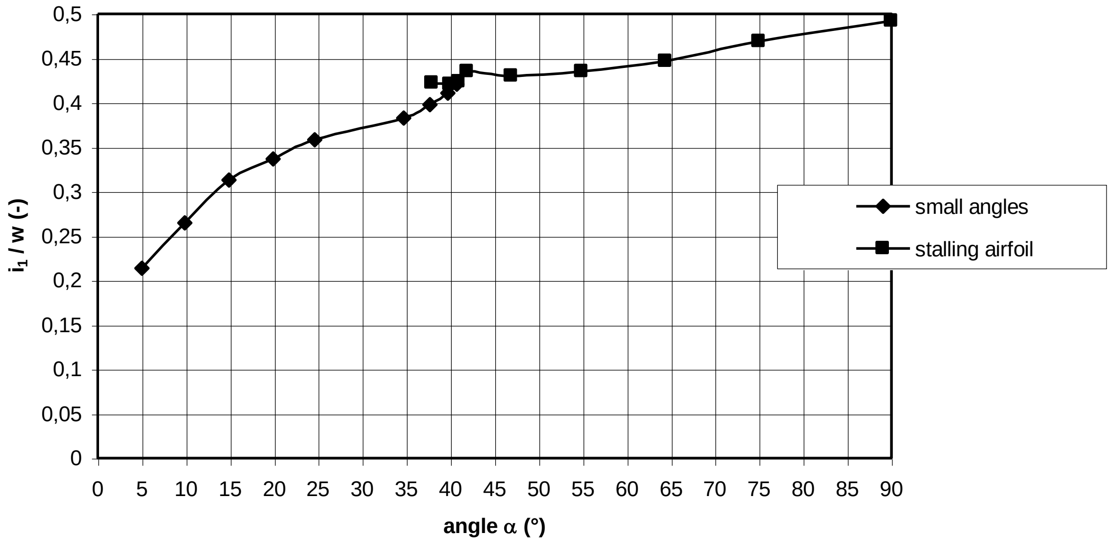

fig. 6 Path of i_1 / w as a function of α using N for a square plate

The angle α will not only vary because of fluctuations in the wind speed but also because of fluctuations in the wind direction. The normal force N is given by:

$N = C_n * ½\rho V^2 * h * w$ (N) (31)

C_n is the normal force coefficient. The C_n-α curve for a square plate is given in figure 6 of KD 213 (ref. 2). This figure is copied as figure 7.

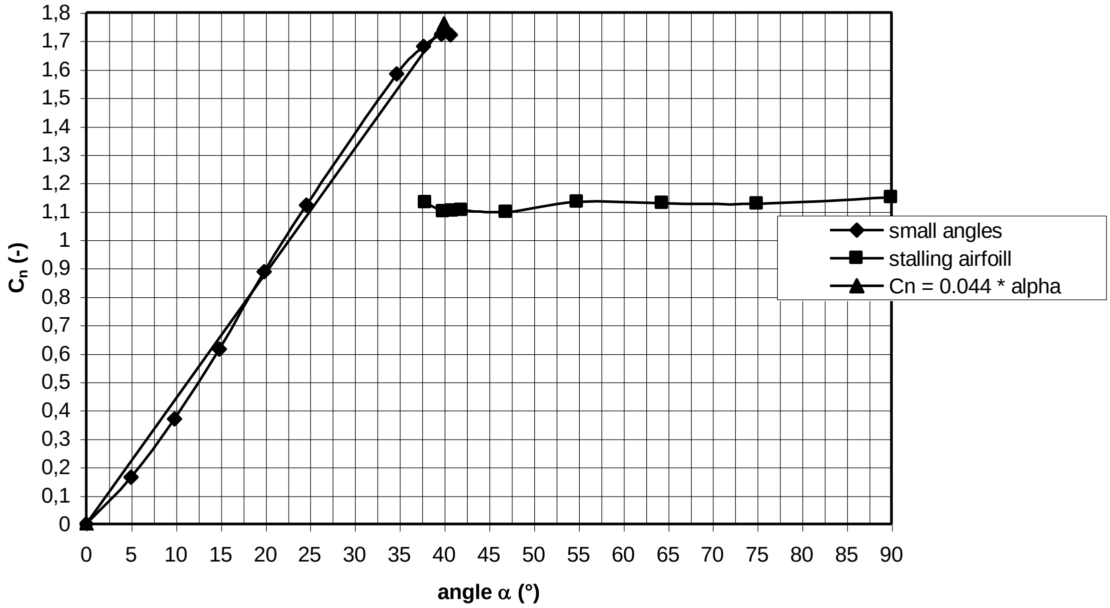

fig. 7 C_n-α curve for a square plate

The C_n-α curve is about a straight line for $0° < \alpha < 40°$. The C_n-α curve for $0° < \alpha < 40°$ can be replaced by the function:

$C_n = 0.044 * \alpha$ (-) $(0° \leq \alpha \leq 40°)$ (32)

This function is also given in figure 7. The angle α is 20° for V = 7 m/s or smaller. This means that sudden variations of the wind direction with a maximum deviation of 20° in both directions result in variation of α in between 0° and 40° with corresponding values of C_n varying in between 0 and 1.76. So the vane will work properly to keep the rotor perpendicular to the wind.

(31) + (32) gives:

$N = 0.044 * \alpha * ½\rho V^2 * h * w$ (N) (33)

(30) + (33) gives:

$M_{vane} = 0.044 * \alpha * ½\rho V^2 * h * w * (R_v + i_1)$ (Nm) (34)

5.3 Determination of M_{spring}

In point f of chapter 4 it was decided that $M_{spring100}$ has the double value as $M_{spring0}$. This means that:

$M_{spring} = M_{spring0} + 1/100\ \gamma * M_{spring0}$ or

$M_{spring} = (1 + 1/100\ \gamma) * M_{spring0}$ (Nm) (35)

M_{spring} will be zero for γ = -100° but this situation can only occur if the stop at zero position is removed. The geometry of the torsion spring has to be chosen such that $M_{spring} = M_{spring0}$ for γ = 0°. Determination of the required torsion spring geometry for which this is realised, is out of the scope of this report. The ratio $i_s = M_{spring} / M_{spring0}$ as a function of γ is given in figure 8.

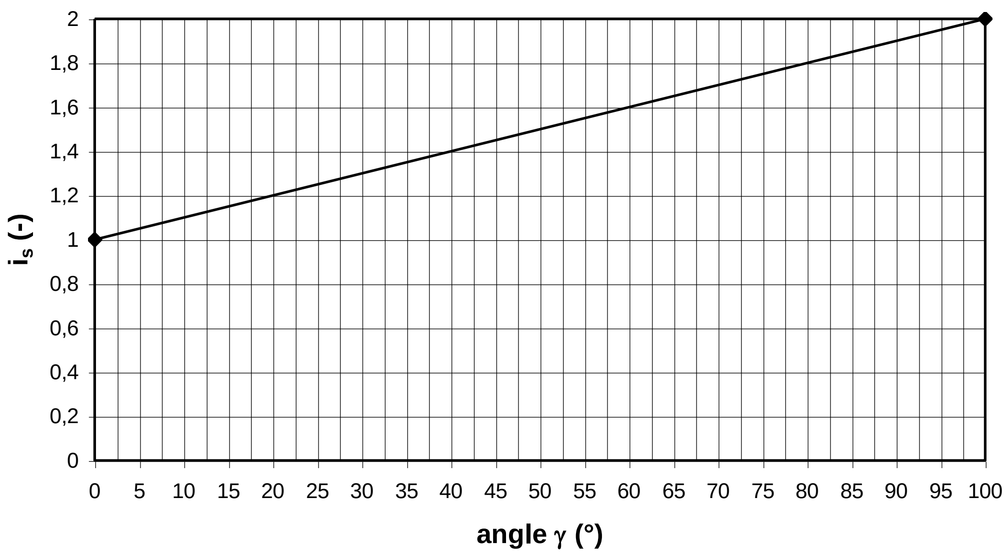

fig. 8 Variation of $i_s = M_{spring} / M_{sprin0}$ as a function of γ

5.4 Determination of the moment equations

(8) + (21) + (34) gives:

$\pi R^3 (C_t * e/R * \cos^2\delta + C_d * f/R * i * \sin\delta - 0.0225 * \sin 3\delta) = 0.044 * \alpha * h * w * (R_v + i_1)$
(for $0° \le \delta \le 40°$) (36)

(8) + (22) + (34) gives:

$\pi R^3 (C_t * e/R * \cos^2\delta + C_d * f/R * i * \sin\delta - 0.0332 * \cos^2\delta) = 0.044 * \alpha * h * w * (R_v + i_1)$
(for $40° \le \delta \le 90°$) (37)

Formula 7 can be written as:

$\gamma = \varepsilon + \delta - \alpha$ (°) (38)

(9) + (21) + (35) + (38) and $\varepsilon = 20°$ gives:

$½\rho V^2 * \pi R^3 (C_t * e/R * \cos^2\delta + C_d * f/R * i * \sin\delta - 0.0225 * \sin 3\delta) =$
$\{1 + 1/100\ (20 + \delta - \alpha)\} * M_{spring0}$ (for $0° \le \delta \le 40°$) (39)

(9) + (22) + (35) + (38) and $\varepsilon = 20°$ gives:

$½\rho V^2 * \pi R^3 (C_t * e/R * \cos^2\delta + C_d * f/R * i * \sin\delta - 0.0332 * \cos^2\delta) =$
$\{1 + 1/100\ (20 + \delta - \alpha)\} * M_{spring0}$ (for $40° \le \delta \le 90°$) (40)

Formulas 36, 37, 39 and 40 are the moment equations. The design wind speed V_d, is the highest wind speed for which the vane arm makes contact with the zero line stop. It is chosen that $V_d = 7$ m/s. So $\delta = 0°$, $\gamma = 0°$ and $\alpha = \varepsilon = 20°$ for $V_d = 7$ m/s (see figure 2). Substitution of $\delta = 0°$ and $\alpha = 20°$ in formula 39 gives:

$M_{spring0} = C_t * ½\rho V^2 * \pi R^2 * e$ (Nm) (for $V = V_d$) (41)

It is assumed that $C_t = 0.7$ (-). The air density $\rho = 1.2$ kg/m^3 for a temperature of 20 °C. $V = V_d = 7$ m/s. Substitution of these values in formula 41 gives:

$M_{spring0} = 20.58 * \pi R^2 * e$ (Nm) (42)

(39) + (42) gives:

$½\rho V^2 * (C_t * \cos^2\delta + C_d * f/e * i * \sin\delta - 0.0225 * R/e * \sin 3\delta) =$
$20.58 * \{1 + 1/100\ (20 + \delta - \alpha)\}$ (for $0° \le \delta \le 40°$) (43)

Suppose we take the rotor and head geometry of the VIRYA-4.2 rotor which is also used in figure 5. Substitution of $\rho = 1.2$ kg/m^3, $C_t = 0.7$, $C_d = 1$, $f/e = 0.48 / 0.42 = 1.1429$, $i = 0.01$ and $R/e = 2.1 / 0.42 = 5$. Substitution of these values in formula 43 gives:

$V^2 * (0.7 * \cos^2\delta + 0.01143 * \sin\delta - 0.1125 * \sin 3\delta) = 34.3 * \{1 + 1/100\ (20 + \delta - \alpha)\}$
(for $0° \le \delta \le 40°$) (44)

Formula 44 can be written as:

$$V = \sqrt{\frac{34.3 * \{1 + 1/100\ (20 + \delta - \alpha)\}}{(0.7 * \cos^2\delta + 0.01143 * \sin\delta - 0.1125 * \sin 3\delta)}} \quad \text{(m/s)} \quad (\text{for } 0° \le \delta \le 40°) \qquad (45)$$

(40) + (42) gives:

$$½\rho V^2 * (C_t * \cos^2\delta + C_d * f/e * i * \sin\delta - 0.0332 * R/e * \cos^2\delta) =$$
$$20.58 * \{1 + 1/100\ (20 + \delta - \alpha)\} \quad (\text{for } 40° \le \delta \le 90°) \qquad (46)$$

Suppose we take the rotor and head geometry of the VIRYA-4.2 rotor which is also used in figure 5. Substitution of ρ = 1.2 kg/m^3, C_t = 0.7, C_d = 1, f/e = 0.48 / 0.42 = 1.1429, i = 0.01 and R/e = 2.1 / 0.42 = 5. Substitution of these values in formula 46 gives:

$$V^2 * (0.7 * \cos^2\delta + 0.01143 * \sin\delta - 0.166 * \cos^2\delta) = 34.3 * \{1 + 1/100\ (20 + \delta - \alpha)\}$$
$$(\text{for } 40° \le \delta \le 90°) \qquad (47)$$

Formula 47 can be written as:

$$V = \sqrt{\frac{34.3 * \{1 + 1/100\ (20 + \delta - \alpha)\}}{(0.534 * \cos^2\delta + 0.01143 * \sin\delta)}} \quad \text{(m/s)} \quad (\text{for } 40° \le \delta \le 90°) \qquad (48)$$

Formula 45 and 48 give the relation in between δ and V for the two δ ranges for 0° ≤ δ ≤ 40° and 40° ≤ δ ≤ 90°. However, a problem with these formulas is that they contain α and that α is a function of V. In figure 2 it can be seen that α = 20° for V = V_d = 7 m/s. Substitution of these values and ρ = 1.2 kg/m^{3}in formula 34 gives:

$$M_{vane} = 25.872 * h * w * (R_v + i) \quad \text{(Nm)} \qquad (49)$$

Formula 49 can be written as:

$$h * w * (R_v + i) = M_{vane} / 25.872 \quad (\text{m}^3) \qquad (50)$$

(34) + (50) gives:

$$V = \sqrt{\{25.872 / (0.044 * \alpha * ½\rho)\}} \qquad (51)$$

Substitution of ρ = 1.2 kg/m^3 in formula 51 gives:

$$V = \sqrt{(980 / \alpha)} \quad \text{(m/s)} \qquad (52)$$

Formula 52 can also be written as:

$$\alpha = 980 / V^2 \quad (°) \qquad (53)$$

Next it will be checked how V is influenced by α. Substitution of α = 20° in formula 52 gives V = 7 m/s, so formula 52 is correct. Substitution of α = 1° in formula 52 gives V = 31.3 m/s. So α becomes very small for high wind speeds and it is allowed to neglect α for high wind speeds.

If α is neglected, formula 45 changes into:

$$V = \sqrt{\frac{34.3 * \{1 + 1/100\,(20 + \delta)\}}{(0.7 * \cos^2\delta + 0.01143 * \sin\delta - 0.1125 * \sin 3\delta)}} \text{ (m/s)} \quad (\text{for } 0° \leq \delta \leq 40°) \; (\text{for } \alpha = 0°) \qquad (54)$$

If α is neglected, formula 48 changes into:

$$V = \sqrt{\frac{34.3 * \{1 + 1/100\,(20 + \delta)\}}{(0.534 * \cos^2\delta + 0.01143 * \sin\delta)}} \text{ (m/s)} \quad (\text{for } 40° \leq \delta \leq 90°) \; (\text{for } \alpha = 0°) \qquad (55)$$

So formulas 54 and 55 are only valid for high wind speeds when it is allowed to neglect α. If it is not allowed to neglect α, one has to use formula 45 and 48.

6 Determination of the δ-V curve for V_d = 7 m/s

It is wanted to determine the δ-V curve for values of δ starting at $\delta = 0°$ and increasing with 10° up to a maximum value $\delta = 80°$. The value $\delta = 64°$ is added too because it appeared that the rotational speed, the thrust and the power have a maximum for $\delta = 64°$. In chapter 5.5 it is explained that the problem of formulas 45 and 48 is that they contain α and that the problem of formula 54 and 55 is that it is only allowed to use them if α can be neglected. This problem is solved by iteration which means that the correct combination of δ and V is approached in some rounds. Five rounds appear to be enough. One has to use the formulas which are valid for a certain δ range so formula 45 and 54 for $0° \leq \delta \leq 40°$ and formula 48 and 55 for $40° < \delta \leq 90°$. Formula 53 is valid for the whole δ range. The procedure of iteration is explained for the δ range $0° \leq \delta \leq 40°$ but it is similar for the other δ range. The following is done:

Round 1
First V is determined for a certain value of δ, using formula 54.
Next α is calculated using formula 53 for the value of V which was found in round 1.
Round 2
Next V is determined for the same value of δ and the angle α from round 1, using formula 45.
Next α is calculated again for the value of V which was found in round 2.
Round 3
Next V is determined for the same value of δ and the angle α from round 2, using formula 45.
Next α is calculated again for the value of V which was found in round 3.
Round 4
Next V is determined for the same value of δ and the angle α from round 3, using formula 45.
Next α is calculated again for the value of V which was found in round 4.
Round 5
Next V is determined for the same value of δ and the angle α from round 4, using formula 45.
Next α is calculated again for the value of V which was found in round 5.

For every new round, the correct combination of δ and V and α is closer approached. The result of the calculations for five rounds is given in table 2. For large angles δ it is even not necessary to use all five rounds because α or V become constant before round 5 is reached.

round 1			round 2			round 3			round 4			round 5		
δ (°)	V(ms)	α (°)	δ (°)	V(ms)	α (°)	δ (°)	V(ms)	α (°)	δ (°)	V(ms)	α (°)	δ (°)	V(ms)	α (°)
0	7	20												
10	8.449	13.73	10	7.990	15.35	10	7.935	15.57	10	7.927	15.60	10	7.926	15.60
20	9.567	10.71	20	9.194	11.59	20	9.163	11.67	20	9.160	11.68	20	9.160	11.68
30	11.092	7.97	30	10.793	8.41	30	10.776	8.44	30	10.775	8.44			
40	13.082	5.73	40	12.845	5.94	40	12.836	5.95	40	12.836	5.95			
50	15.943	3.86	50	15.761	3.95	50	15.757	3.95						
60	20.750	2.28	60	20.618	2.31	60	20.616	2.31						
64	23.644	1.75	64	23.532	1.77	64	23.530	1.77						
70	29.837	1.10	70	29.750	1.11	70	29.749	1.11						
80	50.074	0.39	80	50.026	0.39									

table 2 Calculated relation in between δ and V for V_d = 7 m/s

The combination of δ, V and α found in the highest round is copied in table 3. The corresponding value of γ is calculated using formula 38 and is also given in table 3. The values for wind speeds in between 0 and 7 m/s are also given in table 3. In table 3, the values for cosδ, $\cos^2\delta$, $\cos^3\delta$, V * cosδ, $V^2 * \cos^2\delta$ and $V^3 * \cos^3\delta$ are also given. V * cosδ is an indication for the increase of the rotational speed (see formula 1). $V^2 * \cos^2\delta$ is an indication for the increase of the thrust and the torque (see formula 2 and 3). $V^3 * \cos^3\delta$ is an indication for the increase of the power (see formula 4).

δ (°)	α (°)	γ (°)	V (m/s)	cosδ	$\cos^2\delta$	$\cos^3\delta$	V * cosδ	$V^2 * \cos^2\delta$	$V^3 * \cos^3\delta$
0	20	0	0	1	1	1	0	0	0
0	20	0	1	1	1	1	1	1	1
0	20	0	2	1	1	1	2	4	8
0	20	0	3	1	1	1	3	9	27
0	20	0	4	1	1	1	4	16	64
0	20	0	5	1	1	1	5	25	125
0	20	0	6	1	1	1	6	36	216
0	20	0	7	1	1	1	7	49	343
10	15.60	14.40	7.926	0.9848	0.9698	0.9551	7.806	60.927	475.572
20	11.68	28.32	9.160	0.9397	0.9746	0.8298	8.608	74.091	637.740
30	8.44	41.56	10.775	0.8660	0.75	0.6495	9.331	87.076	812.538
40	5.95	54.05	12.836	0.7660	0.5868	0.4495	9.833	96.687	950.717
50	3.95	66.05	15.757	0.6428	0.4132	0.2656	10.128	102.585	1039.018
60	2.31	77.69	20.616	0.5	0.25	0.125	10.308	106.255	1095.275
64	1.77	82.23	23.530	0.4384	0.1922	0.0842	10.315	106.397	1097.467
70	1.11	88.89	29.749	0.3420	0.1170	0.0400	10.175	103.526	1053.349
80	0.39	99.61	50.026	0.1736	0.0302	0.0052	8.687	75.463	655.538

table 3 Calculated relation in between δ, α, γ and V for V_d = 7 m/s

In table 3 it can be seen that the maximum values are calculated for δ = 64° and not at δ = 80° where M_{spring} has the maximum value. This is mainly caused by $M_{Fs\delta}$ which becomes important at large yaw angles (see figure 5). So the characteristics differ on this point from the pendulum safety system which has a maximum for all four quantities for δ = 70°.

For all values of δ out of table 3 it is assumed that the rotor is loaded such that it rotates with the design tip speed ratio λ_d. For very low wind speeds this is not realistic because the rotor may stand still if it has not yet started. If the generator is charging a battery, the matching in between rotor and generator is normally good for wind speeds above about 4 m/s but for lower wind speeds the rotor is mostly turning at a higher tip speed ratio than λ_d (once the rotor has started). So table 3 is not accurate for wind speeds below about 4 m/s but these wind speeds are not important if the functioning of a safety system is observed.

The calculated values for δ as a function of V are given as the δ-V curve of figure 9.

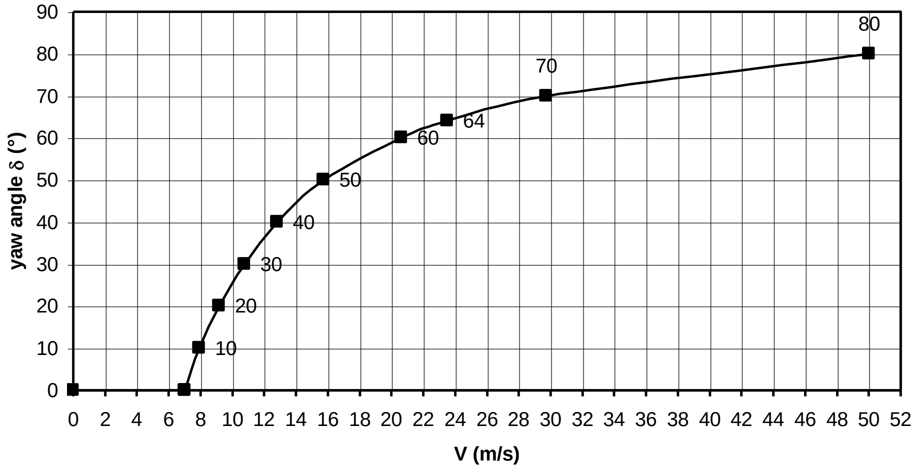

fig. 9 Calculated δ-V curve for the ecliptic safety system with torsion spring for V_d = 7 m/s

If figure 9 is compared to the ideal curve of figure 1, it can be seen that the δ-V curve makes an angle with the V-axis which is smaller than 90°. So the overshoot of the rotational speed and thrust which will happen if the wind varies around V_d = 7 m/s, will be less than for the ideal curve, which is favourable. But the rest of the curve has about the same shape as the ideal curve.

The calculated values of V * cosδ as a function of V are given in figure 10. This curve is an indication of the variation of the rotational speed.

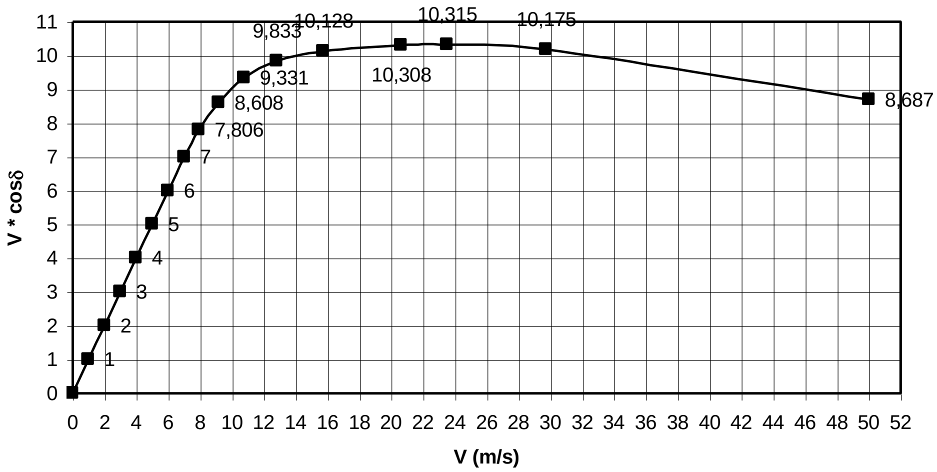

fig. 10 V * cosδ as a function of V for V_d = 7 m/s

In figure 10 it can be seen that the rotational speed is sharply limited and has a maximum at V = 23.53 m/s for δ = 64°. This wind speed is called the rated wind speed V_{rated}, so V_{rated} = 23.53 m/s. The corresponding rotational speed is called n_{rated}. The rotational speed at the design wind speed V_d = 7 m/s is called n_d. The ratio n_{rated} / n_d = 10.315 / 7 = 1.474 which is an acceptable value for V_{rated} / V_d = 23.53 / 7 = 3.361.

The calculated values of $V^2 * \cos^2\delta$ as a function of V are given in figure 11. This curve is an indication of the variation of the thrust and the torque.

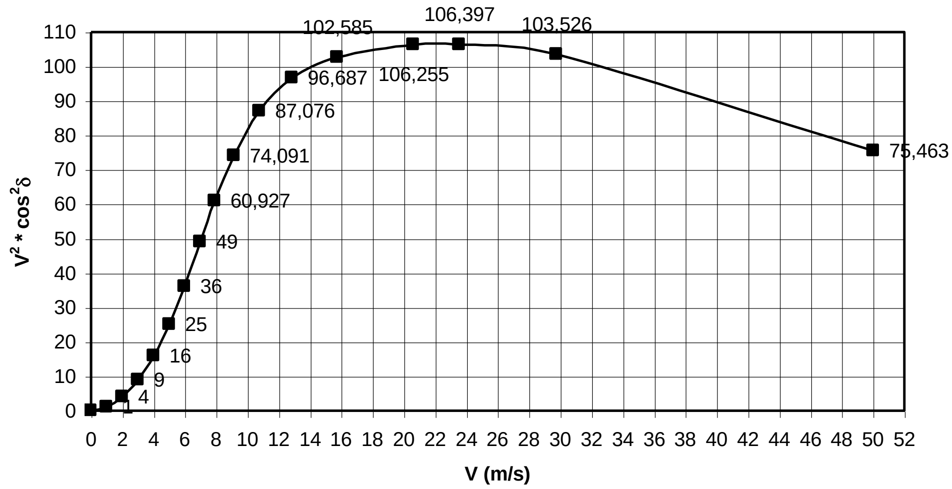

fig. 11 $V^2 * \cos^2\delta$ as a function of V for V_d = 7 m/s

In figure 11 it can be seen that the thrust and the torque are sharply limited and have a maximum at V_{rated} = 23.53 m/s. The ratio in between the rated thrust and the design thrust or in between the rated torque and the design torque is 106.397 / 49 = 2.171. The rotor strength has to be calculated for V_{rated} = 23.53 m/s and δ = 64°.

The calculated values of $V^3 * \cos^3\delta$ as a function of V are given in figure 12. This curve is an indication of the variation of the power.

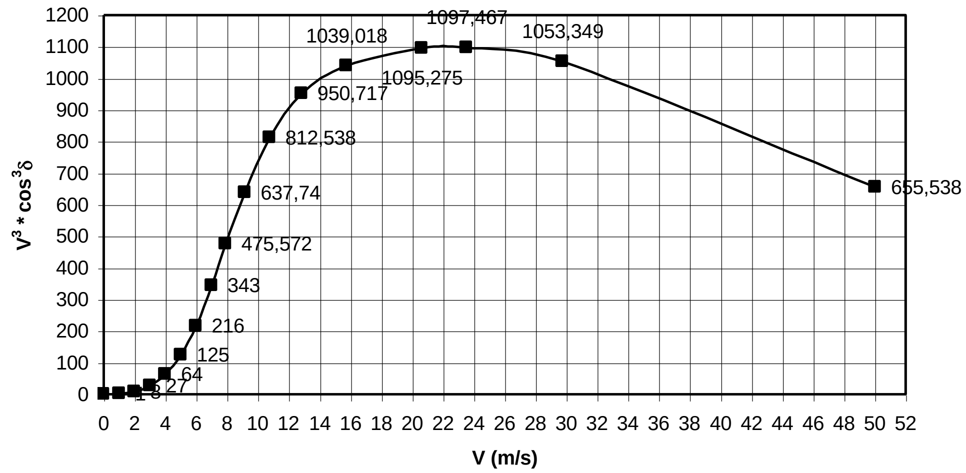

fig. 12 $V^3 * \cos^3\delta$ as a function of V for V_d = 7 m/s

In figure 12 it can be seen that the power is sharply limited and has a maximum at V_{rated} = 23.53 m/s. The ratio in between the rated power and the design power is 1097.467 / 343 = 3.20. The power is the mechanical power supplied by the rotor shaft. For the electrical power, the generator efficiency has to be taken into account.

7 Realisation of a certain design wind speed

In chapter 6 it has been shown that a design wind speed V_d = 7 m/s is a good first choice. For this wind speed, $\delta = 0°$ and $\alpha = 20°$. Substitution of these values in formula 36 gives:

$$C_t * \pi R^2 * e = 0.88 * h * w * (R_v + i_1) \qquad (56)$$

Assume C_t = 0.7. Assume it is chosen that e = 0.2 R. Assume a square vane blade is chosen, so h = w. Assume the average angle α in between the vane blade and the wind direction is 10°. In figure 6 it can be seen that i_1 / w = 0.27 for α = 10°, so i_1 = 0.27 w. Substitution of these values in formula 56 gives:

$$R^3 = 2.0008 * w^2 * (R_v + 0.27\ w) \quad (m^3) \qquad (57)$$

Normally the rotor radius R is chosen first. Next one chooses a square sheet with a side w such that the sheet can be made out of material with standard dimensions such that no or only a little waste material is produced. Next one has to determine R_v such that formula 57 is fulfilled.

R_v is the horizontal length of the vane arm in between the vane axis and the front edge of the vane blade. If the vane arm makes an angle of 45° with the horizon, the real length is a factor √ 2 longer. Next one has to make a composite drawing and check if the vane blade juts above the rotor shadow for the given dimensions of rotor, vane arm and vane blade. If not, one has to increase the length of the vane arm and decrease the width w of the vane blade such that formula 57 is fulfilled.

Once the geometry is OK, one has to determine the construction of the vane hinge and the stops of the vane arm at γ = 0° and at γ = 100°. One also has to determine a torsion spring which has the required characteristics and which supplies $M_{spring0}$ at γ = 0°. The relation in between $M_{spring0}$ and the rotor parameters is given by formula 42. Assume e = 0.2 R. Substitution of this value in formula 42 gives:

$$M_{spring0} = 12.931 * R^3 \quad (Nm) \quad (\text{for } V = V_d = 7\ m/s \text{ and } e = 0.2\ R) \qquad (58)$$

The determination of the geometry of the torsion spring is out of the scope of this report.

8 Use of an auxiliary vane in stead of an eccentric rotor

An eccentrically placed rotor is no problem for an electricity generating windmill because the electricity cable can easily bridge the distance in between the rotor axis and the tower centre. For water pumping windmills it may be wanted that there is no eccentricity but in this case an auxiliary vane has to be used to turn the rotor out of the wind.

If there is no eccentricity, one can use a rectangular gear box if one wants to use a rotating vertical shaft in the tower centre. One must realise that a rotating vertical shaft has a reaction torque which influences the safety system. The influence is lesser if the gear box has a higher accelerating gear ratio. If the windmill is used to drive a rope pump, this pump has almost a constant torque and this means that the rotor will run almost unloaded at high wind speeds. This means that the torque coefficient is very low at high wind speeds and the reaction torque can therefore be neglected at high wind speeds.

An auxiliary vane gives a different behaviour as that of an eccentrically placed rotor. The difference depends on the position of the vane blade and on the aspect ratio i_a. The aspect ratio is the ratio in between the height h of the vane blade and the width w. A square vane blade (with i_a = i) is a good choice for the main vane but a bad choice for a auxiliary vane because the normal coefficient C_n has an instable region at about α = 40° (see figure 7).

Auxiliary vanes are normally placed at the end of a pipe which is positioned at a certain distance behind the rotor plane. The vane blade juts out of the rotor plane. However, this positioning makes that the vane blade comes in the rotor shadow above a certain yaw angle δ. This may be unfavourable because the wind speed in the rotor shadow is not known accurately and because the vane arm can be shaken by the vortex coming from the blade tip. So in the first instance it is assumed that the pipe has a forward bend at the end which brings the vane blade in the rotor plane. The distance in between the side of the vane blade and the rotor tip is chosen at least 25 % of the rotor radius R. It is assumed that the vane blade is in the undisturbed wind speed V. It is also assumed that the vane blade is parallel to the rotor plane. So the angle of attack α = 90° when the yaw angle δ = 0°. The C_n-δ curve is therefore the mirror image of the C_n-α curve.

Next an aspect ratio is chosen for which the characteristic of the auxiliary vane is approaching the characteristic of an eccentrically placed rotor as good as possible. The rotor thrust decreases proportional to $\cos^2\delta$ (see formula 2). A $\cos^2\delta$ function is given in figure 13.

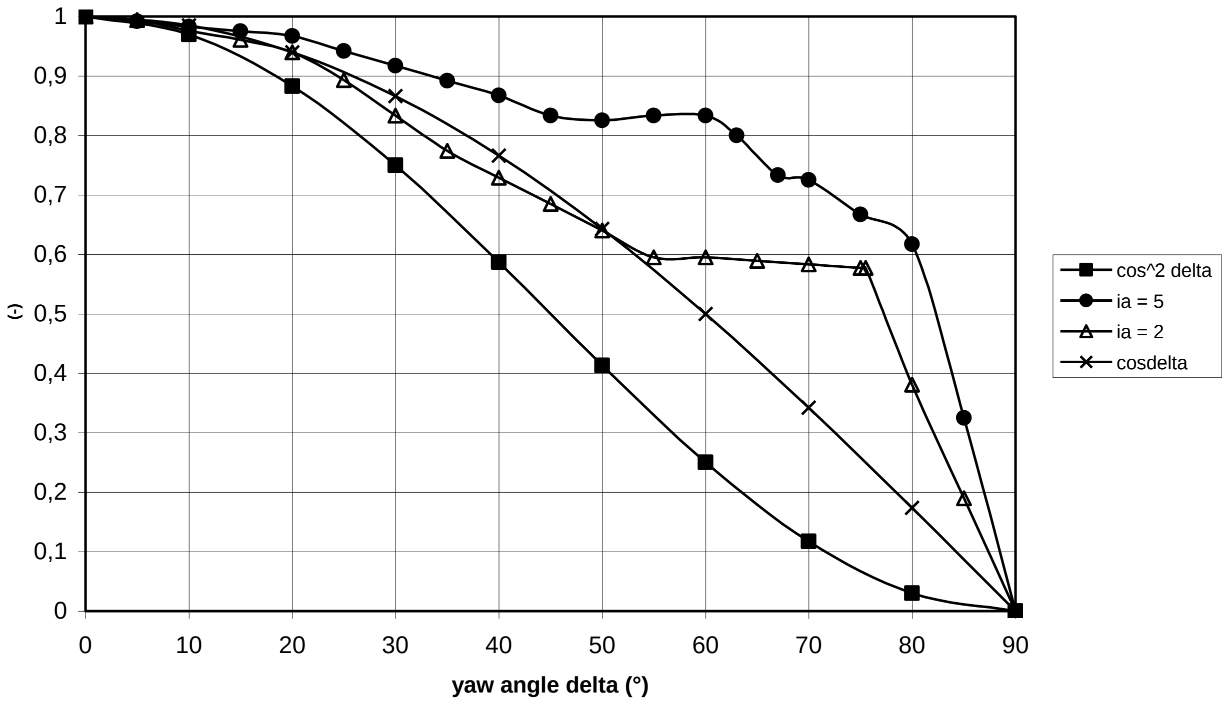

fig. 13 $\cos^2\delta$, i_a = 5, i_a = 2 and $\cos\delta$

It is assumed that the length of the vane arm and the size of the vane blade are chosen such that the moment of the vane blade around the tower axis is the same as the moment produced by the thrust of an eccentrically placed rotor for δ = 0°. This means that the shape of the M-δ curve of the auxiliary vane is congruent to the mirror image of the C_n-α curve of the vane blade. This also means that it is 1 for δ = 0° if it is compared to the $\cos^2\delta$ curve of an eccentrically placed rotor.

The C_n-α curves for five different aspect ratios i_a = 5, i_a = 2, i_a = 1, i_a = 1/2 and i_a = 1/5 are given in figure 13 and figure 31 of report R 999 D (ref. 1). The C_n-α curves for i_a = 1/2 and i_a = 1/5 have instable regions just as for i_a = 1, so only the curves for i_a = 5 and i_a = 2 are taken into account. These C_n-α curves are given in figure 14.

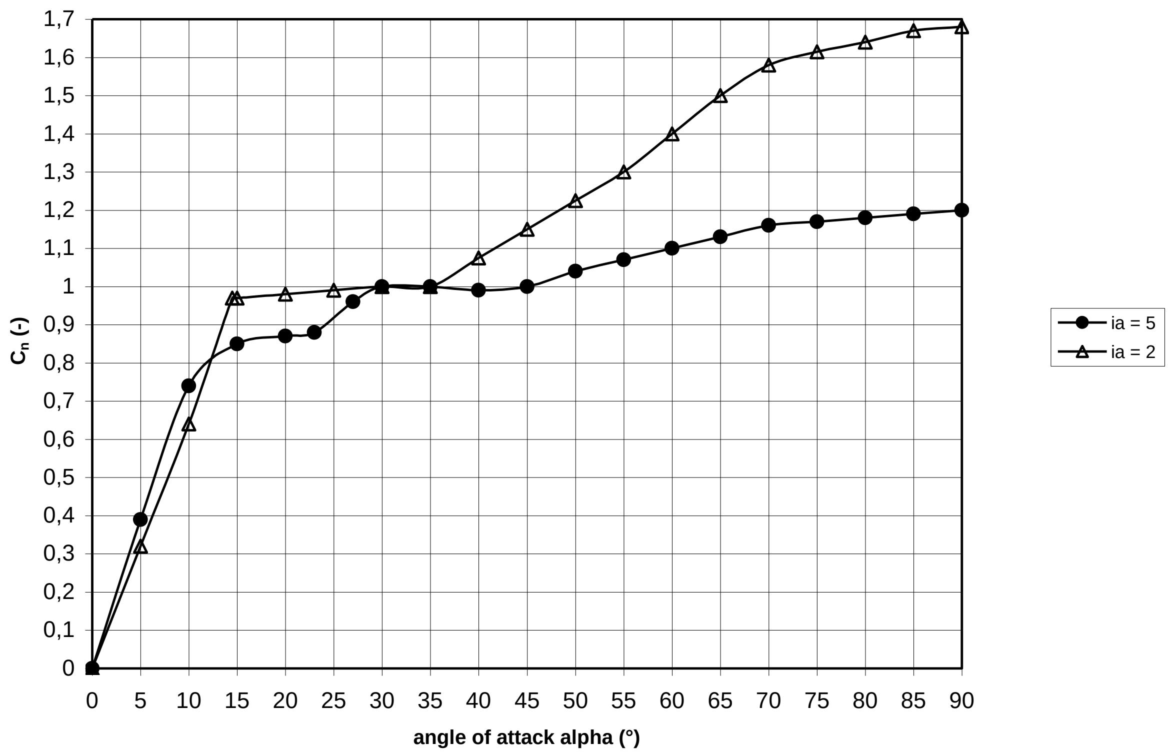

fig 14 C_n-α curves for $i_a = 5$ and $i_a = 2$

In figure 14 it can be seen that both curves are almost the same for small angles of α but that the curve for $i_a = 2$ is much higher for large angles of α. So a smaller vane blade area is needed for $i_a = 2$ if a certain force is needed at low wind speeds for which $\delta = 0°$ and so for which $\alpha = 90°$.

The curves of figure 14 are now mirror imaged around the vertical axis and reduced by a factor such that the value for $\alpha = 90°$, and so for $\delta = 0°$, is 1. Next, these mirrored curves are drawn in figure 13.

In figure 13 it can be seen that the curve $i_a = 5$ is lying much higher than the curve for $\cos^2\delta$. This means that, if the windmill is provided with an auxiliary vane with $i_a = 5$, it will turn much more out of the wind than for an eccentric rotor if the rotor is perpendicular to the wind at low wind speeds. It is safe but it will result in a much stronger reduction of the power than needed.

In figure 13 it can be seen that the curve $i_a = 2$ is also lying higher than the curve for $\cos^2\delta$ but the difference is less than for $i_a = 5$. So a vane blade with an aspect ration $i_a = 2$ is a better choice. The difference in between both curves is still rather high for yaw angles larger than 50° so the rotor will turn out of the wind too much for high wind speeds. This effect may be compensated by conscious positioning of the vane blade behind the rotor plane and so it will be in the rotor shadow for large yaw angles. The rotor shadow results in reduction of the wind speed at the vane blade and therefore in reduction of the moment produced by the vane blade around the tower axis. The elimination of the bend in the vane arm simplifies the construction. It has to be tested in practice if this works nice and if the vane arm isn't shaking.

Another way to reduce the vane moment at large yaw angles is to use a cylindrical vane blade with the cylinder axis vertical. Only a drag force is working on a cylinder in the direction of the wind. The moment of a cylindrical vane blade therefore varies according to a cosine. A $\cos\delta$ curve is also given in figure 13.

The cosδ curve is lying close to the curve $i_a = 2$ for $0 < \delta < 55°$ but for higher values of δ it is lying considerably lower.

The drag coefficient C_{dva} for a cylindrical pipe depends on the Reynolds value and on the pipe roughness but is 1.18 for a smooth pipe for Reynolds smaller than 10^5. The relation is given by a rather complex figure as figure 54 in report R 999 D (ref. 1). Only the line for a smooth pipe is copied as figure 15. In the original graph the x-axis and the y-axis are both logarithmic. However, a logarithmic graph is very difficult to read accurately. Therefore the axis of figure 15 are made linear. The values for C_{dva} for Reynolds larger than $2.2 * 10^5$ are not given in figure 54 of R 999 D but are estimated copying the shape of the curves for higher roughness.

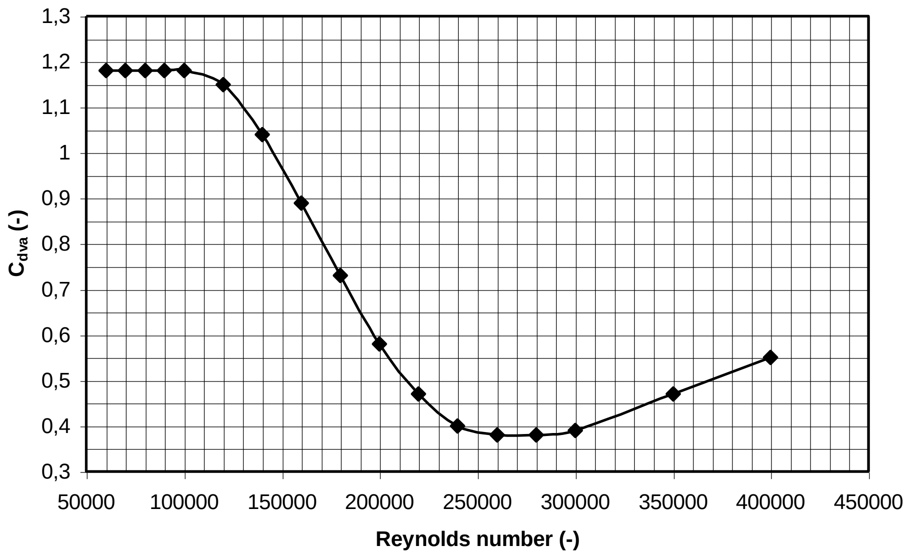

fig. 15 Path of C_{dva} as a function of the Reynolds number for a smooth pipe

The Reynolds value Re is calculated using formula 71 from R 999 D. This formula is copied as formula 59:

$$Re = V * d / \nu \qquad (-) \tag{59}$$

In this formula V is the wind speed (m), d is the pipe diameter (m) and ν is the kinematic viscosity which is about $15 * 10^{-6}$ m^2/s for air. Because of the reduction of the drag coefficient at high wind speeds, the vane moment will be reduced at high wind speeds. So the real shape of the curve at large angles δ will lie lower than the cosδ curve given in figure 13. This means that the real M-δ curve is approaching the $\cos^2\delta$ curve even better than a cosδ curve. So an auxiliary vane with a cylindrical vane blade is the best option if the safety system can have no eccentricity.

An eccentrically placed rotor is always better than even a cylindrical vane blade because the swept area of the rotor is much bigger than the area of a vane blade and the generated force is therefore less sensible to local fluctuations of the wind speed caused by turbulence.

The length of the vane arm and the geometry of the vane blade is chosen such that the same moment around the tower axis is found for δ = 0° as for an eccentrically placed rotor with e = 0.2 R for δ = 0°. The relation in between the main vane geometry and the rotor radius for a square main vane is given by formula 56 which is valid if the rotor is perpendicular to the wind. A similar formula can be derived for the auxiliary vane. First one has to choose a certain vane geometry. Assume a flat auxiliary vane blade with an aspect ratio $i_a = 2$ is chosen.

The width of the auxiliary vane is called w_{av}. The height of the auxiliary vane is called h_{av}. As $i_a = 2$ it is valid that $h_{av} = 2 * w_{av}$. The length of the arm of the auxiliary vane is called R_{av}. The value of R_{av} must be chosen such that the vane blade juts outside of the rotor. It is chosen that $R_{av} = 1.25 * R$. The normal force acting on the auxiliary vane blade is called N_{av}. This normal force is acting at a distance i_2 from the inner edge of the vane blade. Just as for a square vane blade, this distance i_2 depends on the angel of attack α (see figure 6). However, the vane blade is calculated for $\delta = 0°$ so for $\alpha = 90°$ for which $i_2 = ½\ w_{av}$. So the moment of the auxiliary vane M_{av} around the tower axis is given by:

$M_{av} = N_{av} * (R_{av} + ½\ w_{av})$ (Nm) (60)

The normal force on the auxiliary vane is given by:

$N_{av} = C_n * ½\rho V^2 * h_{av} * w_{av}$ (N) (61)

Substitution of $h_{av} = 2 * w_{av}$ in formula 61 gives:

$N_{av} = C_n * ½\rho V^2 * 2\ w_{av}^2$ (N) (62)

(60) + (62) gives:

$M_{av} = C_n * ½\rho V^2 * 2\ w_{av}^2 * (R_{av} + ½\ w_{av})$ (Nm) (63)

If the vane blade is perpendicular to the wind, the value of C_n is 1.2 for a vane blade with $i_a = 2$ (see figure 14 for $\alpha = 90°$). Substitution of $C_n = 1.2$ in formula 63 gives:

$M_{av} = 2.4 * ½\rho V^2 * w_{av}^2 * (R_{va} + ½\ w_{av})$ (Nm) (64)

This moment must be equal to the moment of an eccentricity placed rotor with an eccentricity $e = 0.2\ R$ if the rotor is perpendicular to the wind, so for $\delta = 0°$. This gives:

$M_{av} = M_{rotor}$ (65)

The rotor moment is given by formula 21. The side force and the self orientating moment are both zero for $\delta = 0°$. So for this condition formula 21 changes into:

$M_{rotor} = ½\rho V^2 * \pi R^2 * C_t * e$ (Nm) (66)

(64) + (65) + (66) gives:

$2.4 * w_{av}^2 * (R_{va} + ½\ w_{av}) = \pi R^2 * C_t * e$ (67)

Substitution of $R_{va} = 1.25$ R, $C_t = 0.7$ and $e = 0.2$ R in formula 67 gives:

$2.4 * w_{av}^2 * (1.25\ R + ½\ w_{av}) = 0.14 * \pi R^3$ (68) or

$5.4567\ w_{av}^2 * (1.25\ R + ½\ w_{av}) = R^3$ (69)

This is a third degree equation and it is difficult to write w_{av} as a function of R. w_{av} can be found by try and error. It is found that equation 69 becomes an equality for $w_{av} = 0.3581$ R.
So the vane blade of the auxiliary vane must have a width $w_{av} = 0.3581$ R and a height $h_{av} = 0.7162$ R. The length of the vane arm $R_{av} = 1.25$ R. This is a rather large vane.

The use of an auxiliary vane in stead of an eccentrically placed rotor might be the best option for a water pumping windmill driving a piston pump by means of a pump rod in the hart of the tower or by driving a rope pump by means of a rotating vertical axis coupled to the rotor axis by an accelerating rectangular gear box. The rotor of such windmill, the VIRYA-2.4 with a diameter of 2.4 m and a design tip speed ratio λ_d = 2.5, is described in report KD 487 (ref. 7).

The self orientating moment of rotors with a low design tip speed ratio is low or even negative. A scale model of the VIRYA-2.4 rotor with a diameter of 0.5 m has been made recently and it was found that the self orientating moment was almost zero as the rotor turns out of the wind even if the eccentricity is zero. This means that a much smaller eccentricity is allowed than e = 0.2 R and I think that e = 0.1 R will still give a stable safety system. The area of the main vane and the auxiliary vane will get about half the area as for e = 0.2 R. This means that the vane width and vane height will be about a factor $\sqrt{2}$ smaller and this results in a more elegant design. The torsion spring in the main vane will also be much smaller.

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