The 7.14 %, 10 % and 12.5 % cambered plate as airfoil for windmill rotor blades.
Aerodynamic characteristics, geometry, moment of inertia I and moment of resistance W.

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1 Introduction

In report KD 35 (ref. 1) a method is given for the design of a windmill rotor. The blade of the rotor must be provided with an aerodynamic airfoil. In the examples in chapter 5.4 of KD 35 the Gö 623 airfoil is used. This is a good airfoil for small wind turbines and it can be used for wooden blades. It is used in the VIRYA-2.2, -3, -4.2 and -4.6 windmills developed by Kragten Design (KD). However wooden blades are rather difficult to manufacture, need to be painted very well and are sensible for damage during transport. Therefore KD is developing a range of small windmills using galvanised steel or stainless steel sheet for the rotor blades. The design tip speed ratios of these windmills are lying in between 3.5 and 6. All rotors have two or three blades with cambered plate airfoils. In figure 4.6 and 4.7 of KD 35 it can be seen that the airfoil must have a C_d/C_l value of at least about 0.04 to realise a $C_{p\ th}$ of 0.4.

The minimum C_d/C_l value of a cambered plate depends on the rate of cambering. The camber C is the airfoil thickness a (not the plate thickness t) divided by the chord c multiplied by 100 %. 10 % cambering is very common for rotor blades used in water pumping windmills with design tip speed ratios in between 1 and 2. The 10 % cambered plate has been measured by different institutes and many measurements are given in the form of graphs and tables in report R443D (lit. 2). However, the minimum C_d/C_l lays in between 0.045 and 0.07 depending on the Reynolds value. This is rather high for rotors with design tip speed ratios in between 3.5 and 6 and this requires a cambered plate with lesser cambering.

In report KD 96 (ref. 3) all information is given about the 7.14 % cambered airfoil. However, this report contains hand written graphs and pictures and can therefore not be sent by e-mail. Sometimes one wants to compare the 7.14 % cambered airfoil with the 10 % and 12.5 % cambered airfoil and the characteristics of these airfoils are therefore also included in this new report KD 398 which replaces the old report KD 96.

2 The 7.14 % cambered plate airfoil characteristics

The 7.14 % cambered plate has only been measured by Imperial College and the measurements are given only as C_l-α and C_l-C_d curves at page 3-6 of R443D. Unfortunately, the measuring points are not given in tables like it is done for many other airfoils. But the graphs are drawn accurately and the measuring points can be copied from the graphs. The C_d-α and the C_m-α curves are not given however, the C_d-α curve can be derived from the C_l-α and C_l-C_d curves. The airfoil has been measured for the Reynolds values (Re) $1.2 * 10^5$, $1.7 * 10^5$, $2.5 * 10^5$ and $3.4 * 10^5$. The plate thickness t is given as 1.63 mm. However, the chord has not been given but it is expected to be 100 mm. The minimum C_d/C_l value for the 7.14 % cambered plate is lying in between 0.03 and 0.04 depending on Re, which is low enough to use this airfoil for rotors with rather high tip speed ratios. However, report R343D is no longer available and it was not possible to find the original graphs of Imperial College. Therefore the graphs given at page 3-6 of report R343D are copied and given as figures 1 and 2 in this report KD 398. The C_l-α curves for larger values of Re are given only for a small α range. The C_l-C_d curve for Re = $1.7 * 10^5$ is lying right from the C_l-C_d curve for Re = $1.2 * 10^5$ which is strange because normally the drag D increases at decreasing Re.

The C_d/C_l ratio for a certain point on C_l/C_d curve can be determined by drawing a line through this point and the origin. The C_d value of the point of intersection of this line with the horizontal line for $C_l = 1$, gives the C_d/C_l ratio. A line through the origin which is touching the C_l-C_d curve has the minimum C_d/C_l ratio. The touching point is called the optimum C_l value.

For the calculation of the starting torque coefficient $C_{q\ start}$, as given by formula 6.12 of KD 35, C_l values for large angles α are needed. However, the 7.14 % cambered plate has only been measured up to $\alpha = 20°$. For large angles α the airfoil is completely stalling and therefore the rate of cambering will have only a limited effect on the C_l-α and C_d-α curves. It might be acceptable to use the curve for a 10 % cambered plate, given in figure 5.

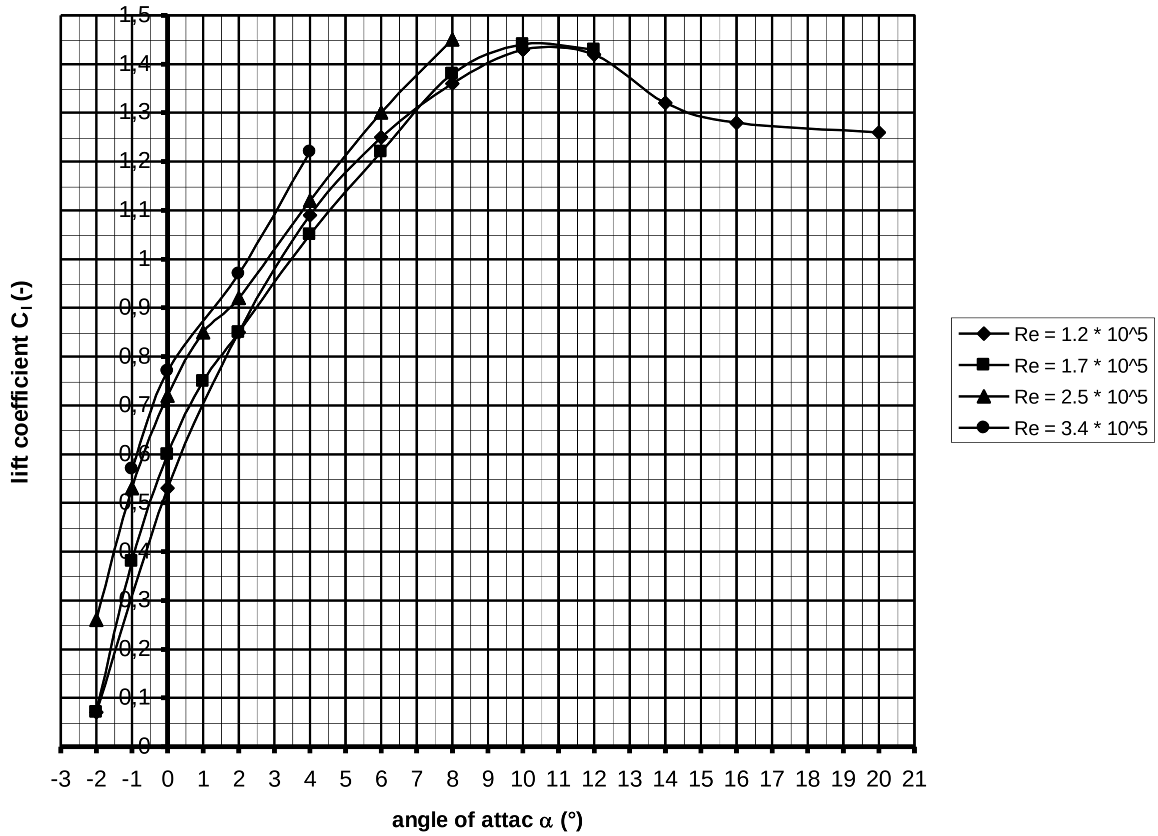

fig. 1 C_l-α curves for 7.14 % cambered plate for four Reynolds values

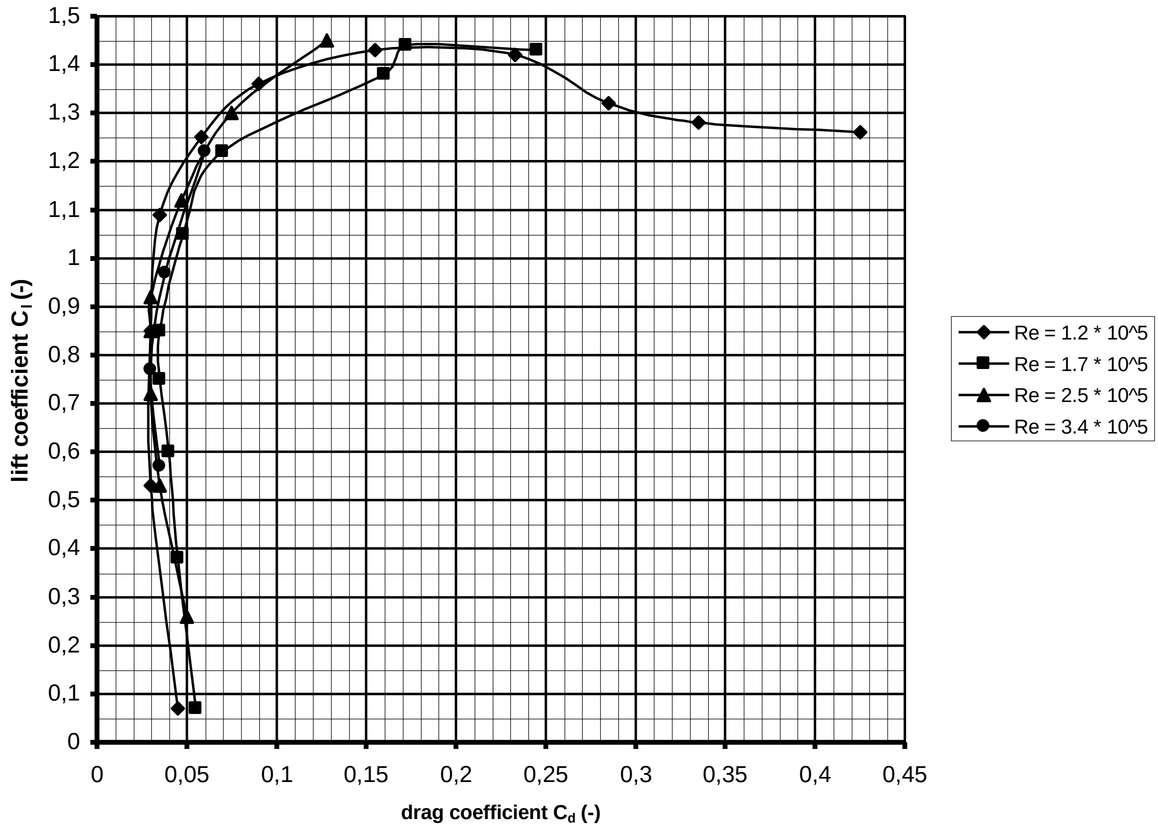

fig. 2 C_l-C_d curves for 7.14 % cambered plate for four Reynolds values

3 The 10 % cambered plate airfoil characteristics

The 10 % cambered plate has been measured by Imperial College, by Volkers and by Bruining (of Delft University of Technology). The measurements of Imperial College are given only as C_l-α and C_l-C_d curves at page 3-7 of R443D. The minimum C_d/C_l value for the 10 % cambered plate is lying in between 0.04 and 0.06 depending on Re, which is substantial higher than for the 7.14 % cambered plate. The C_d/C_l ratio determines the maximum C_p (see chapter 4.3 of KD 35) and the 10 % cambered plate can therefore only be used for rotors with rather low design tip speed ratios.

The measurements of Volkers are given on page 3-11 of R443D. The measurements of Bruining are given on page 3-12 up to 3-18 of R443D. The measuring points of Bruining are also given in tables on page 3-41 up to 3-44 of R443D. If the measurements of Imperial College, Volkers and Bruining are compared it can be seen that there is some difference. This must be caused by the use of different wind tunnels and measuring equipment. Because I want to compare airfoils with different camber, I have used the measurements of Imperial College for the normal α region. The C_l-α curves, which are copied from page 3-7 of R443D, are given as figure 3. The C_l/C_d curves are given as figure 4.

For the calculation of the starting torque coefficient $C_{q\ start}$, as given by formula 6.12 of KD 35, C_l values for large angles α are needed. However, the 10 % cambered plate has only been measured by Imperial College up to α = 20° for Re = 1.2 * 10^5. Therefore, for the large angles α, the measurements of Bruining have been used. At starting, the Reynolds value is rather low and the measuring points for Re = 1 * 10^5 have been used which are given in the table at page 3-42 of R443D. The C_l-α curve for large angles α is given in figure 5. The measurements of Imperial College for Re = 1.2 * 10^5 are used for -2° < α < 20°. The measurements of Bruining for Re = 1.2 * 10^5 are used for -2° < α < 90°. It can be seen that there is some difference in between the curves. This might be caused by the different Reynolds value or by different wind tunnel turbulence or different measuring equipment.

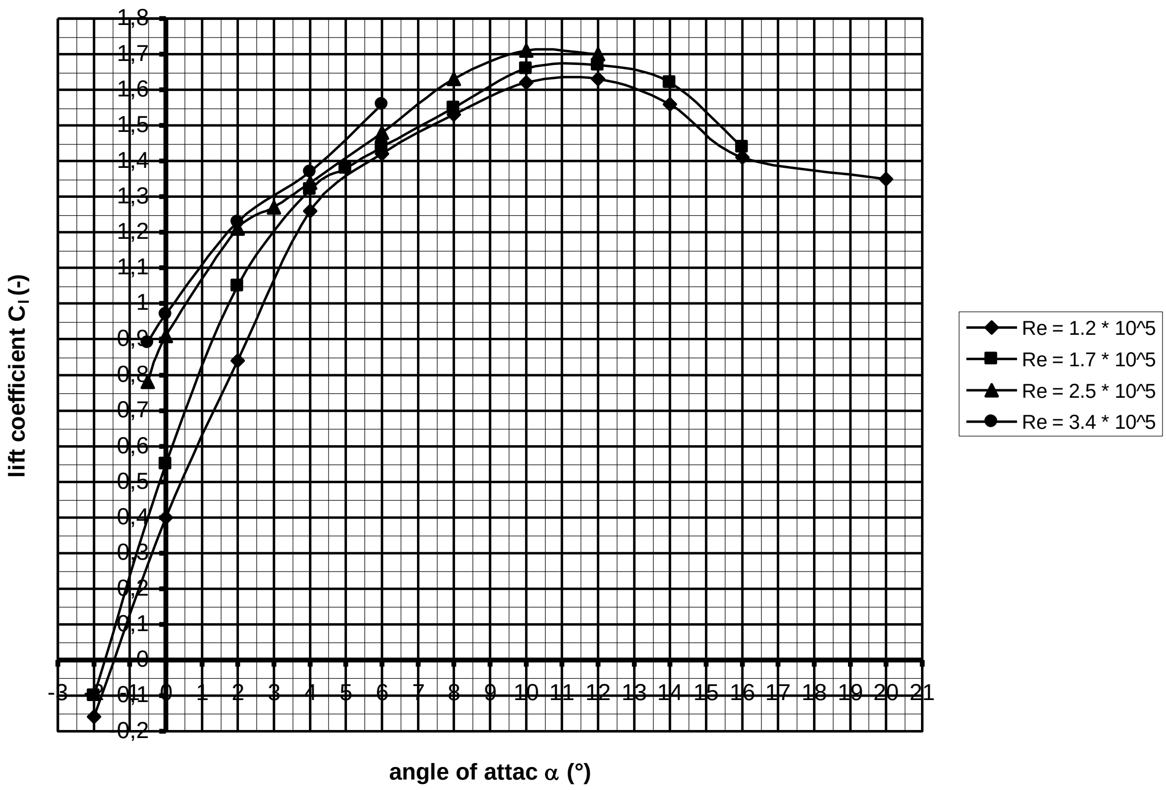

fig. 3 C_l-α curves for 10 % cambered plate for four Reynolds values

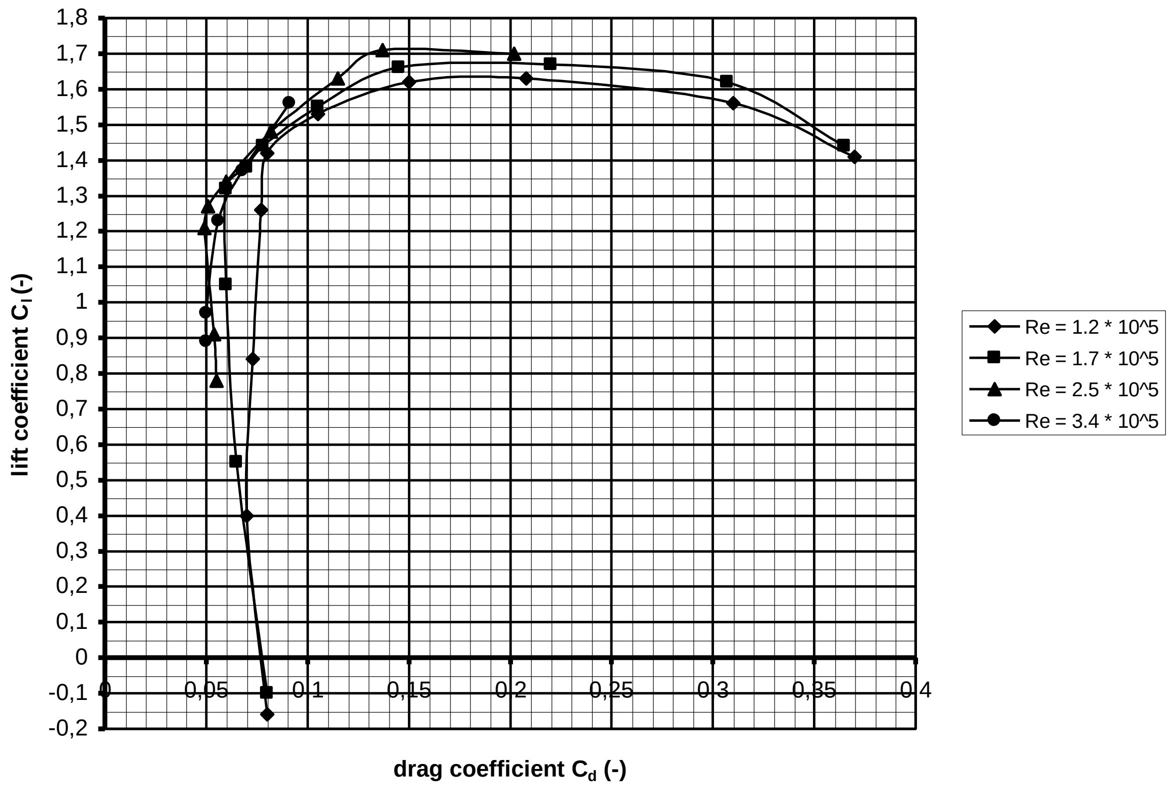

fig. 4 C_l-C_d curves for 10 % cambered plate for four Reynolds values

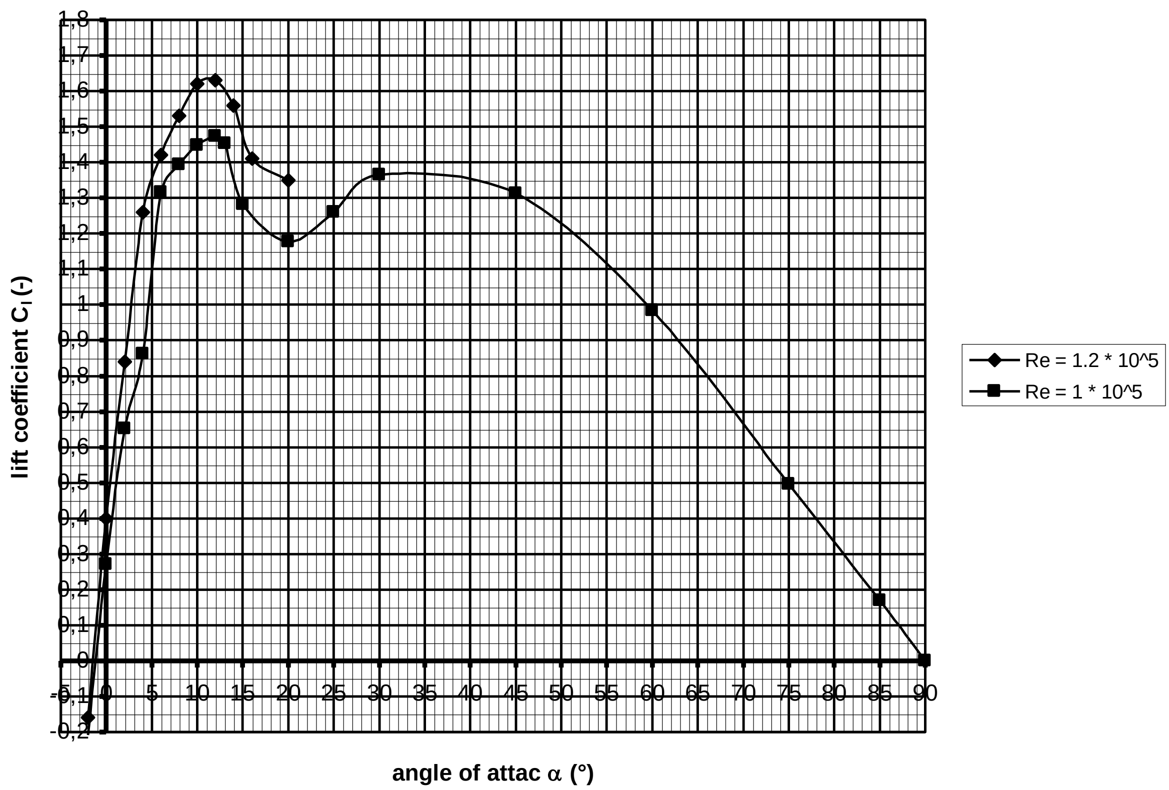

fig. 5 C_l-α curves for 10 % cambered plate for large angles α. Measurements of $1.2 * 10^5$ from Imperial College. Measurements of $1 * 10^5$ from Bruining.

4 The 12.5 % cambered plate airfoil characteristics

The 12.5 % cambered plate has only been measured by Imperial College. The measurements of are given only as C_l-α and C_l-C_d curves at page 3-8 of R443D. The minimum C_d/C_l value for the 12.5 % cambered plate is lying in between 0.055 and 0.07 depending on Re, which is substantial higher than for the 10 % and the 7.14 % cambered plate. The C_d/C_l ratio determines the maximum C_p (see chapter 4.3 of KD 35) and the 12.5 % cambered plate can therefore only be used for rotors with rather low design tip speed ratios.

The C_l-α curves, which are copied from page 3-8 of R443D, are given as figure 6. The C_l/C_d curves are given as figure 7. The C_l-C_d curve is not given for Re = 3.4 * 10^5 in R443D so it can't be copied.

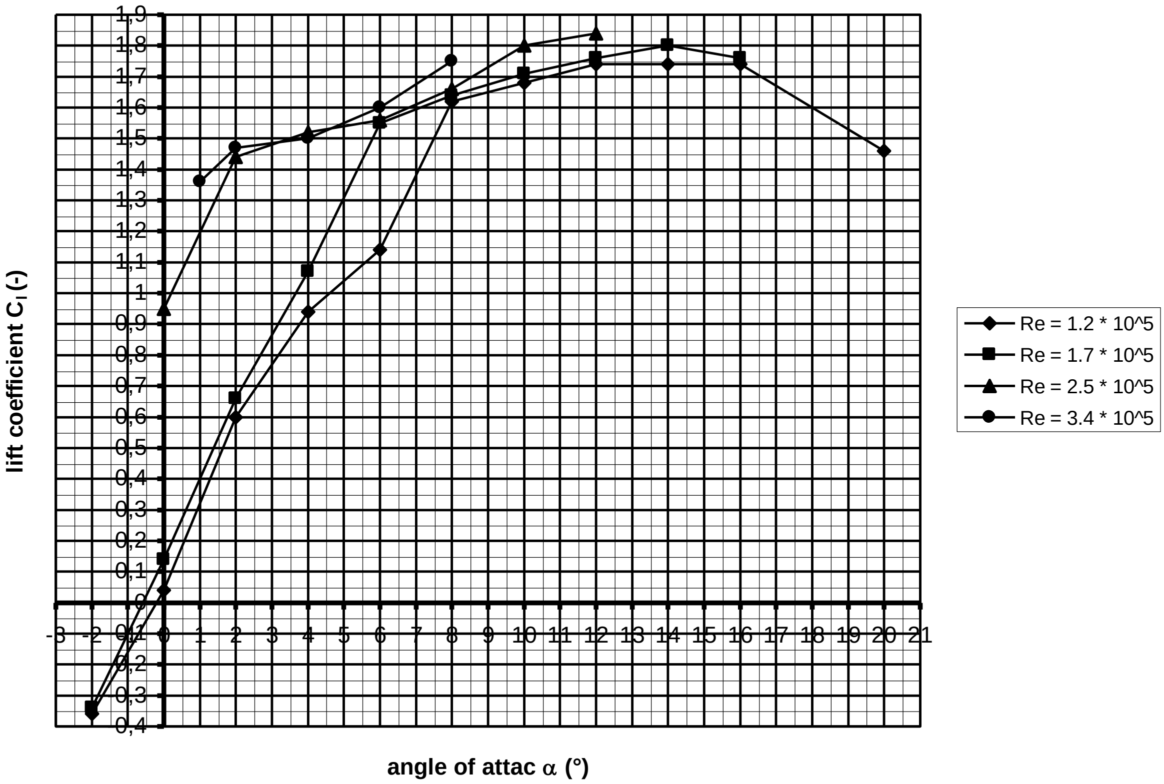

fig. 6 C_l-α curves for 12.5 % cambered plate for four Reynolds values

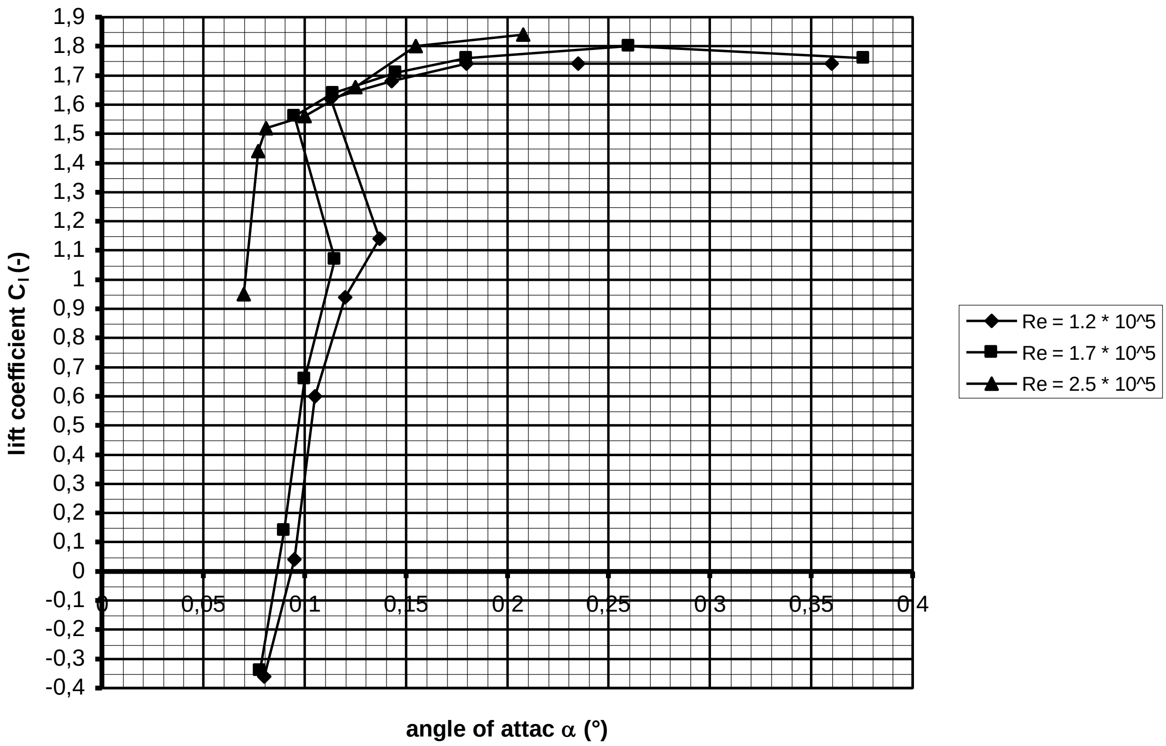

fig. 7 C_l-C_d curves for 12.5 % cambered plate for three Reynolds values

If the C_l-α curves of fig. 1, fig. 3 and fig. 6 for different camber are compared, it can be seen that the highest lift coefficients can be realised for the highest camber. A higher lift coefficient means that a smaller chord c is required to generate a certain lift L. However, especially the curves for 12.5 % camber are very sensitive for the Reynolds value and much higher angles α are needed to realise a certain lift coefficient for a low Reynolds number than for a high one. A high angle α results in a smaller blade angle β for a certain angle ϕ in between the relative wind direction and the rotor plane. A smaller blade angle β results in a lower starting torque coefficient.

If the C_l-C_d curves of fig. 2, fig. 4 and fig. 7 for different cambers are compared, it can be seen that the lowest C_d/C_l ratio can be realised for the lowest camber. A low C_d/C_l ratio result in a high maximum power coefficient C_p (see report KD 35 chapter 4.3.3). The C_l-C_d curves for 12.5 % camber are also very sensitive for the Reynolds value and have a strong discontinuity (the measuring points of fig. 6 and 7 are therefore connected by straight lines in stead of curved lines for the other five figures). This means that a low C_d/C_l ratio is only possible for a certain high optimum C_l value. If the C_l value is higher or lower than the optimum C_l value, the C_d/C_l ratio is very much lower than for the optimum C_l value.

Windmill rotors can be designed for the optimum C_l value and in this case they will have the maximum C_p for the given airfoil. However, this results in a rotor with a varying chord and blade angle. Windmill rotors can also be designed for a constant chord but this results in an increasing lift coefficient for decreasing radius (see report KD 35 chapter 5.4.2). But variation in the lift coefficient is only acceptable for airfoils which have a good C_d/C_l ratio for a large C_l range and so for a large α range. The 7.14 % cambered airfoil is very good on this point but the 12.5 % cambered airfoil is very bad. To my opinion a 12.5 % airfoil should never be used for a rotor with constant chord blades. For rotors with a high design tip speed ratio, only a 7.14 % cambered airfoil has an acceptable low C_d/C_l value for an acceptable large α range. For rotors with a design tip speed ratio up to about 3 one can use a 10 % cambered airfoil. However, if the airfoil is disturbed by a supporting pipe, one has the use the characteristics of a 10 % cambered plate with a pipe (also measured by Bruining).

5 Determination of the cambered plate airfoil geometry

A cambered plate airfoil is made from a plate with width b. The chord c and the airfoil thickness a depend on the cambering rate. The cambering radius r_c is important for manufacture. The relevant magnitudes are given in figure 8.

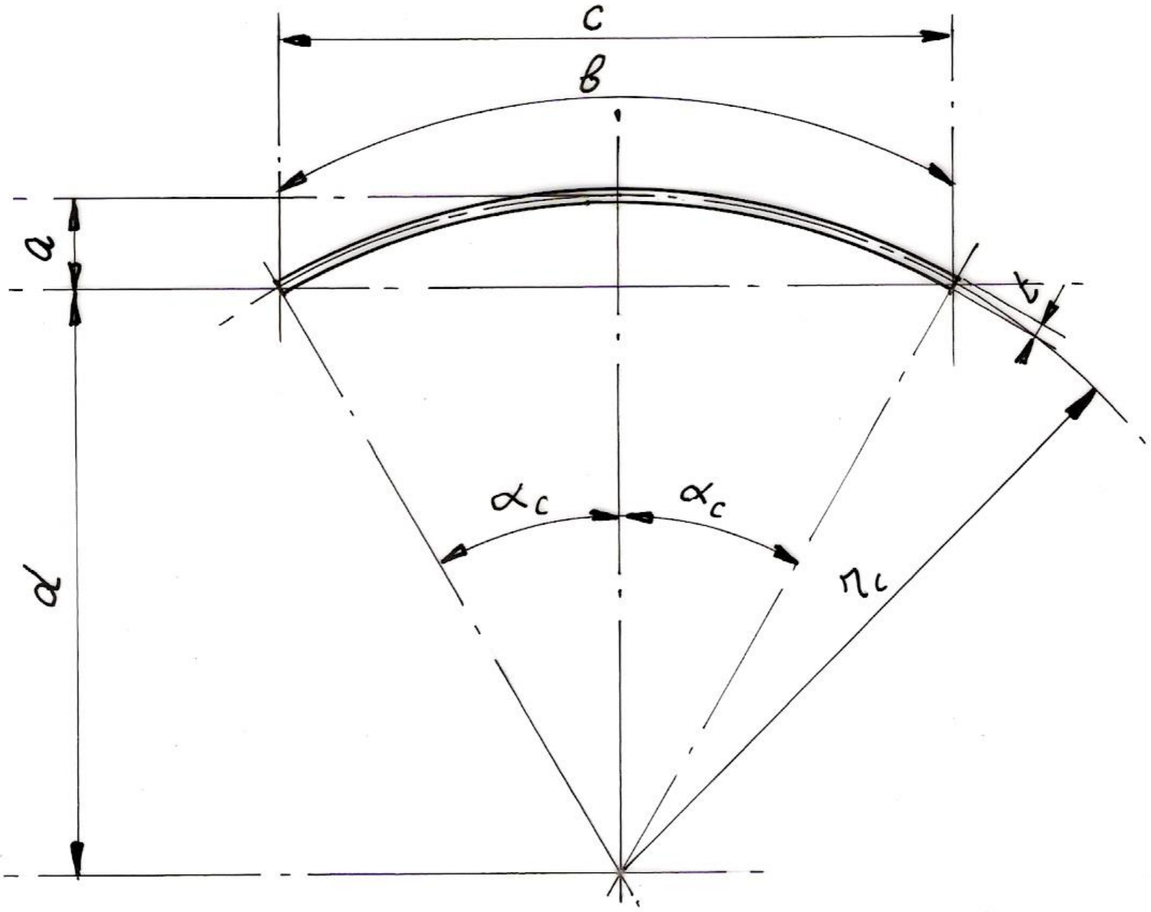

fig. 8 Geometry cambered plate airfoil

The expressions for a, c, the cambering angle α_c (in radian) and the camber C for a certain plate width b and a certain cambering radius r_c is found as follows.

$b / 2 \pi r_c = 2 \alpha_c / 2 \pi$ or

$\alpha_c = b / 2 r_c$ (α_c in radian) (1)

$a = r_c – d$ (mm) (2)

$d = r_c * \cos\alpha_c$ (mm) (3)

(2) + (3) gives:

$a = r_c (1 - \cos\alpha_c)$ (mm) (α_c in radian) (4)

$½ c / r_c = \sin\alpha_c$ or

$c = 2 r_c * \sin\alpha_c$ (mm) (α_c in radian) (5) camber $C = a/c * 100$ (%) (6)

(4) + (5) + (6) gives:

$$C = \frac{1 - \cos\alpha_c}{2 * \sin\alpha_c} * 100 \quad (\%) \quad (\alpha_c \text{ in radian}) \qquad (7)$$

Formulas 1, 4, 5 and 7 can be used to calculate α_c, a, c and C if b and r_c are given (the calculator has to be set in the mode "radian"). However, it is very difficult to find expressions for α_c, a, c and r_c if b and C are given like this is the practical situation for a cambered plate. This problem can be solved by estimating different values for r_c and calculating the camber C till the correct camber is gained. This is done for a width b = 100 mm for the three different cambers. The following result is gained:

5.1 camber C = 7.14 %

b = 100 mm
r_c = 176 mm
α_c = 0.2840909 radian
a = 7.055 mm
c = 98.660 mm
a/c = 0.0715 so C = 7.15 %

For other widths b, the values for a, c and r_c vary proportional with the scale factor.

5.2 camber C = 10 %

b = 100 mm
r_c = 126.5 mm
α_c = 0.3952569 radian
a = 9.753 mm
c = 97.416 mm
a/c = 0.1001 so C = 10.01 %

For other widths b, the values for a, c and r_c vary proportional with the scale factor.

5.3 camber C = 12.5 %

b = 100 mm
r_c = 102 mm
α_c = 0.4901961 radian
a = 12.011 mm
c = 96.043 mm
a/c = 0.1251 so C = 12.51 %

For other widths b, the values for a, c and r_c vary proportional with the scale factor.

6 Determination of the moment of inertia I and the moment of resistance W

The magnitudes which are important for the bending moment of inertia I which determines the bending stiffness and the moment of resistance W which determines the bending strength are given in figure 9.

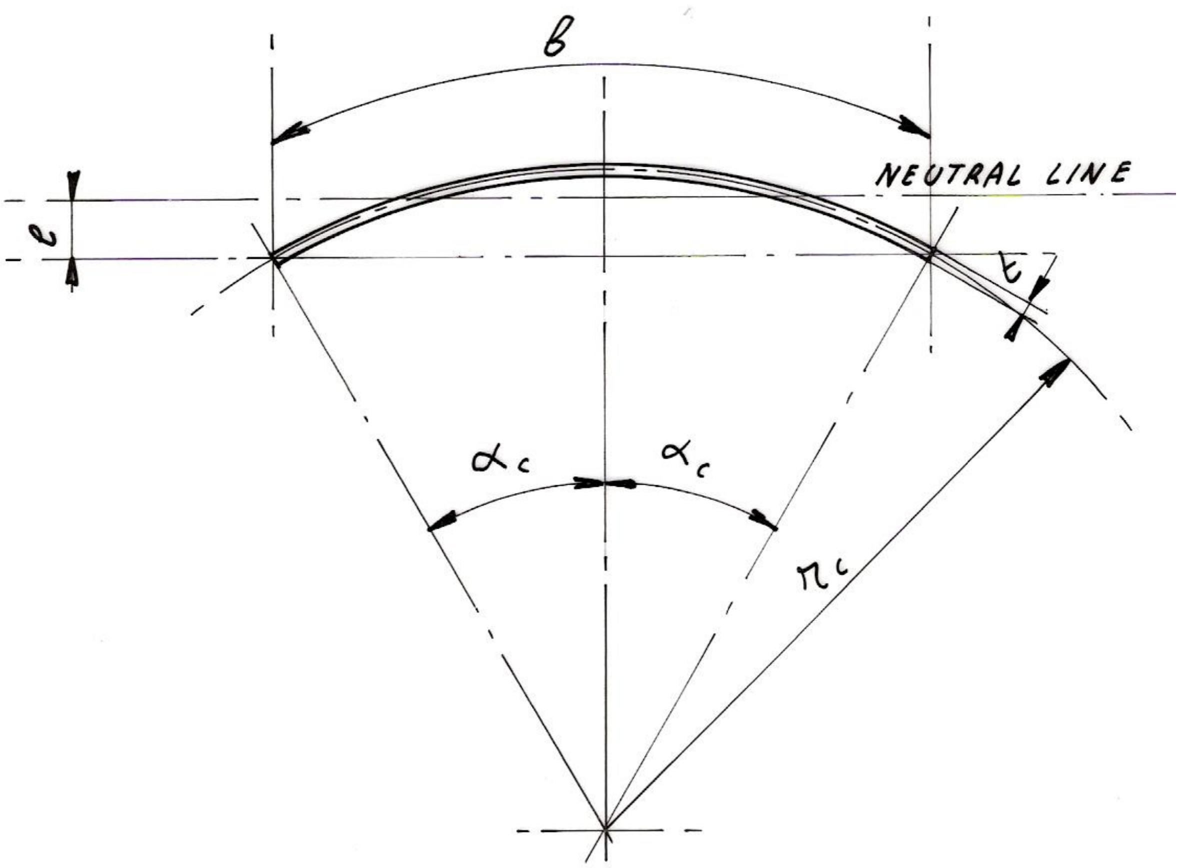

fig. 9 Cambered plate airfoil and position of the neutral line

The bending moment of inertia I is given by:

$$I = r_c^3 * t\ (\alpha_c + \sin\alpha_c * \cos\alpha_c - 2/\alpha_c * \sin^2\alpha_c) \quad (mm^4) \quad (\alpha_c \text{ in radian}) \qquad (8)$$

α_c has to be calculated very accurate because the term in between brackets is very small.

The distance e from the neutral line till the lower side of the airfoil is given by:

$$e = r_c\ (1/\alpha_c * \sin\alpha_c - \cos\alpha_c) \quad (mm) \quad (\alpha_c \text{ in radian}) \qquad (9)$$

The moment of resistance W is I / e so:

$$W = \frac{r_c^2 * t\ (\alpha_c + \sin\alpha_c * \cos\alpha_c - 2/\alpha_c * \sin^2\alpha_c)}{(1/\alpha_c * \sin\alpha_c - \cos\alpha_c)} \quad (mm^3) \quad (\alpha_c \text{ in radian}) \qquad (10)$$

6.1 camber C = 7.14 %

Substitution of α_c = 0.2840909 radian in formula 8 gives:

$I = 0.000081301\ r_c^3 * t \quad (mm^4)$ (11)

Substitution of α_c = 0.2840909 radian in formula 10 gives:

$W = 0.0030466\ r_c^2 * t \quad (mm^3)$ (12)

6.2 camber C = 10 %

Substitution of α_c = 0.3952569 radian in formula 8 gives:

$I = 0.00041929\ r_c^3 * t \quad (mm^4)$ (13)

Substitution of α_c = 0.3952569 radian in formula 10 gives:

$W = 0.0081786\ r_c^2 * t \quad (mm^3)$ (14)

6.3 camber C = 12.5 %

Substitution of α_c = 0.4901961 radian in formula 8 gives:

$I = 0.0012155\ r_c^3 * t \quad (mm^4)$ (15)

Substitution of α_c = 0.4901961 radian in formula 10 gives:

$W = 0.015545\ r_c^2 * t \quad (mm^3)$ (16)

6.4 Example VIRYA-1.8 blade

Formulas 11 and 12 are now used to calculate I and W for the VIRYA-1.8 blade which has a plate width b = 125 mm, a thickness t = 2 mm and 7.14 % camber.

The plate width is 125 mm, so the scale factor is 125 / 100 = 1.25. This results in r_c = 1.25 * 176 = 220 mm. Substitution of r_c = 220 mm and t = 2 mm in formula 11 gives I = 1731.4 mm^4. The moment of inertia I of a flat plate with b = 125 mm and t = 2 mm is 1/12 * 125 * 2^3 = 83.33 mm^4. So the 7.14 % camber results in an increase of I, and so in an increase of the bending stiffness, with a factor 1731.38 / 83.33 = 20.77. Be alert, this factor is only constant for a ratio of b / t = 125 / 2 = 62.5 and for 7.14 % camber.

The torsion stiffness doesn’t increase by the camber. This is the reason why cambered plate airfoils are rather sensible for an aerodynamic instability called flutter, if the plate thickness is taken too small.

Substitution of r_c = 220 mm and t = 2 mm in formula 12 gives W = 294.9 mm^3. The moment of resistance W of a flat plate with b = 125 mm and t = 2 mm is 1/6 * 125 * 2^2 = 83.33 mm^3. So the 7.14 % camber results in an increase of W, and so in an increase of the bending strength, with a factor 294.9 / 83.33 = 3.54. Be alert, this factor is only constant for a ratio of b / t = 125 / 2 = 62.5 and for 7.14 % camber.

7 References

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3 Kragten A. The 7.14 % cambered plate as airfoil for windmill rotor blades, Aerodynamic characteristics, geometry, moment of inertia I and moment of resistance W, July 2002, modified July 2008, report KD 96, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.