Calculations executed for the 3-bladed rotor of the VIRYA-1.04 windmill (λ_d = 3.5, 7.14 % cambered, aluminium blades) meant to be coupled to a Nexus hub dynamo

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1 Introduction

The VIRYA-1.04 windmill is developed for manufacture in western as well as in developing countries. The VIRYA-1.04 has a 3-bladed rotor with aluminium blades which are directly bolted to the front flange of a Shimano Nexus hub dynamo type DH-2R40 or another type with similar flange and shaft dimensions. The hub dynamo is normally used in the front wheel of a bicycle where it generates a nominal AC voltage of 6 V and a nominal electrical power of 2.4 W. However, for windmill use, the 1-phase alternating current is rectified by a bridge rectifier and the rotational speed and so the voltage, are increased such that a 12 V lead acid battery can be charged. The maximum power is about 6 W at a maximum charging voltage of 14 V. The maximum current will be about 0.43 A and it is expected that no voltage controller will be needed if a 12 V battery of about 30 Ah is used.

The VIRYA-1 will be equipped with the hinged side vane safety system which is also used for all other VIRYA windmills. However, the head will be designed such that it can be made without welding. The head pipe and the tower pipe will both be made out of a 1 m long ½” stainless steel gas pipe. The tower pipe can be connected to a wooden pole by two stainless steel clamping blocks and four threaded rods M6. The vane blade will be made of 1.5 mm aluminium sheet. It is expected that the rated wind speed is 8 m/s for this vane blade.

2 Description of the rotor of the VIRYA-1.04 windmill

The 3-bladed rotor of the VIRYA-1.04 windmill has a diameter D = 1.04 m and a design tip speed ratio λ_d = 3.5. Advantages of a 3-bladed rotor are that the gyroscopic moment is not fluctuating and that a 3-bladed rotor looks nicer than a 2-bladed or 4-bladed rotor.

The rotor has blades with a constant chord and is provided with a 7.14 % cambered airfoil. A blade is made of an aluminium strip with dimensions of 1.5 * 125 * 500 mm. 32 blades can be made out of a standard sheet of 1 * 2 m, 50 blades can be made out of a standard sheet of 1.25 * 2.5 m and 72 blades can be made out of a standard sheet of 1.5 * 3 m without waste material. For cambering of the blades, a simple blade press is designed. Tools for twisting the blades and for pressing the head bearings in the tower pipe, are also designed.

The hub dynamo has two identical flanges. For connection of the spokes, each flange has 18, 2.6 mm holes at a pitch angle of 20° and at a pitch circle diameter of 80 mm. The flange at the side of the electricity cable is called the back flange and the other flange is called the front flange. For use in a bicycle, the back flange side of the dynamo has to be mounted at the right side of the bicycle to realise the correct direction of rotation of the hub. The rotor is connected to the front flange and this means that the rotor must rotate left hand to realise the correct direction of rotation of the hub. The front flange has a collar with a diameter of 64 mm. A moon shaped excision is made in the blade root for centring on the collar.

In the front flange, nine spoke holes at a pitch angle of 40° are modified into 4 mm holes. Each blade is connected to the front dynamo flange by three stainless steel bolts, washers and self locking nuts M4. As there is a distance of 20 mm in between the hart of the dynamo shaft and the blade root, the radius R at the blade tip is 520 mm = 0.52 m. So the diameter D = 1.04 m resulting in the name VIRYA-1.04 for this windmill. There is a certain overlap of the blades at the blade root and this overlap is used to connect the three blades together with three stainless steel bolts, washers and self locking nuts M4.

The outer part of a blade is 7.14 % cambered over a length of 400 mm. This part is twisted linear right hand seen from the tip. The chord c is a little smaller than the strip width because of the camber and this results in c = 123.3 mm = 0.1233 m. The inner part of a blade is flat over a length of 30 mm. The remaining 70 mm is used for the transition of flat to camber and is twisted left hand to realise the correct blade setting angle at cross section E. All four corners of a blade are rounded with r = 5 mm.

The mass of all three blades together is about 0.76 kg which is very light for a rotor with a diameter of 1.04 m. It is expected that it is not necessary to balance the rotor if the hole pattern at the blade root is made accurately. It might be necessary to develop a drilling jig to realise this. A sketch of the rotor is given in appendix 1. Detailed drawings of rotor, head, tower pipe and overall assembly have been made and are available in a separate manual (ref. 5).

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 1). This report (KD 518) has its own formula numbering. Substitution of λ_d = 3.5 and R = 0.52 m in formula (5.1) of KD 35 gives:

$\lambda_{r\,d} = 6.7308 * r$ (-) (1)

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$\beta = \phi - \alpha$ (°) (2)

$\phi = 2/3 \arctan 1 / \lambda_{r\,d}$ (°) (3)

Substitution of B = 3 and c = 0.1233 m in formula (5.4) of KD 35 gives:

$C_l = 67.945\, r\, (1 - \cos\phi)$ (-) (4)

Substitution of V = 5 m/s and c = 0.1233 m in formula (5.5) of KD 35 gives:

$R_{e\,r} = 0.411 * 10^5 * \sqrt{(\lambda_{r\,d}^2 + 4/9)}$ (-) (5)

The blade is calculated for five stations A till E which have a distance of 0.1 m of one to another. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l, α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The aerodynamic characteristics of a 7.14 % cambered airfoil are given in report KD 398 (ref. 2). The Reynolds values for the stations are calculated for a wind speed of 5 m/s because this is a reasonable wind speed for a windmill which is designed for a rated wind speed of 9 m/s. Those airfoil Reynolds numbers are used which are lying closest to the calculated values. The calculated Reynolds values for V = 5 m/s are rather low and so the lowest available Reynolds value Re = 1.2 * 10^5 has to be used for all five stations.

station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\,th}$ (-)	$C_{l\,lin}$ (-)	$R_{e\,r} * 10^{-5}$ V = 5 m/s	$R_e * 10^{-5}$ 7.14 %	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	$C_d/C_{l\,lin}$ (-)
A	0.52	3.5	10.6	0.1233	0.61	0.80	1.46	1.2	0.5	1.6	10.1	9.0	0.035
B	0.42	2.827	13.0	0.1233	0.73	0.70	1.19	1.2	1.2	1.0	11.8	12.0	0.039
C	0.32	2.154	16.6	0.1233	0.91	0.80	0.93	1.2	2.5	1.6	14.1	15.0	0.035
D	0.22	1.481	22.7	0.1233	1.16	1.16	0.67	1.2	4.7	4.7	18.0	18.0	0.035
E	0.12	0.808	34.0	0.1233	1.40	1.37	0.43	1.2	9.0	13.0	25.0	21.0	0.19

table 1 Calculation of the blade geometry of the VIRYA-1.04 rotor

The theoretical blade angle β_{th} varies in between 10.1° and 25.0°. If a blade angle of 9° is taken at the blade tip and of 21° at station section E, the linearised blade angles are lying close to the theoretical values for the most important outer part of the blade. So the blade is twisted 12° right hand in between station A and E. The transition part of the blade is twisted 21° left hand to get the correct blade angle at station section E.

4 Determination of the C_p-λ and the C_q-λ curves

The determination of the C_p-λ and C_q-λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.036. Figure 4.7 of KD 35 (for B = 3) en λ_{opt} = 3.5 and C_d/C_l = 0.036 gives $C_{p\ th}$ = 0.435 (interpolation in between the lines for C_d/C_l = 0.03 and C_d/C_l = 0.05). The blade is stalling at station E and therefore not the whole cambered blade length k = 0.4 m is used for the calculation of $C_{p\ max}$ but only the part up to 0.05 m outside station E. This gives k' = 0.35 m.
Substitution of $C_{p\ th}$ = 0.435, R = 0.52 m and blade length k = k' = 0.35 m in formula 6.3 of KD 35 gives $C_{p\ max}$ = 0.39. $C_{q\ opt} = C_{p\ max} / \lambda_{opt}$ = 0.39 / 3.5 = 0.1114.
Substitution of $\lambda_{opt} = \lambda_d$ = 3.5 in formula 6.4 of KD 35 gives λ_{unl} = 5.6.
The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - ½k) * C_l * c * k / \pi R^3 \qquad (-) \qquad (6)$$

The average blade angle β = 15°. For a non rotating rotor, the angle ϕ = 90°. The average angle of attack α is therefore 90° - 15° = 75°. The estimated C_l-α curve for large values of α is given as figure 5 of KD 398. For α = 75° it can be read that C_l = 0.5. During starting, the whole blade is stalling, so now the whole cambered blade length k = 0.4 m is used for the calculation of $C_{q\ start}$.
Substitution of B = 3, R = 0.52 m, k = 0.4 m, C_l = 0.5 en c = 0.1233 m in formula 6 gives that $C_{q\ start}$ = 0.0402. The real starting torque coefficient is a little lower than the calculated value because we have used the average blade angle and the average angle of attack. Assume $C_{q\ start}$ = 0.038. For the ratio in between the starting torque and the optimum torque we find that it is 0.038 / 0.1114 = 0.34. This is rather high for a rotor with a design tip speed ratio of 3.5.

The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * ½\rho * \pi R^3}\right)} \qquad (m/s) \qquad (7)$$

The peak of the sticking torque Q_s of the Nexus hub dynamo has been measured and it was found that Q_s = 0.084 Nm. Substitution of Q_s = 0.084 Nm, $C_{q\ start}$ = 0.038, ρ = 1.2 kg/m^3 and R = 0.52 m in formula 7 gives that V_{start} = 2.9 m/s. The average sticking torque Q_{sa} at very low rotational speeds has also been measured and it was found that Q_{sa} = 0.054 Nm. Substitution of Q_{sa} = 0.054 Nm, $C_{q\ start}$ = 0.038, ρ = 1.2 kg/m^3 and R = 0.52 m in formula 7 gives that V_{start} = 2.3 m/s. So this means that once the rotor is rotating a little, it will start at a wind speed of 2.3 m/s because of the fly wheel effect of the rotor. So the effective starting wind speed will be about 2.6 m/s. This is acceptable for a low wind regime.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p-λ and C_q-λ curves can be determined if only two points of the C_p-λ curve and one point of the C_q-λ curve are known. The first part of the C_q-λ curve is determined according to KD 35 by drawing an S-shaped line which is horizontal for λ = 0.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 3). With this method, it can be determined that the C_q-λ curve is directly rising for low values of λ if a 7.14 % cambered sheet airfoil is used. This effect has been taken into account and the estimated C_p-λ and C_q-λ curves for the VIRYA-1.04 rotor are given in figure 1 and 2.

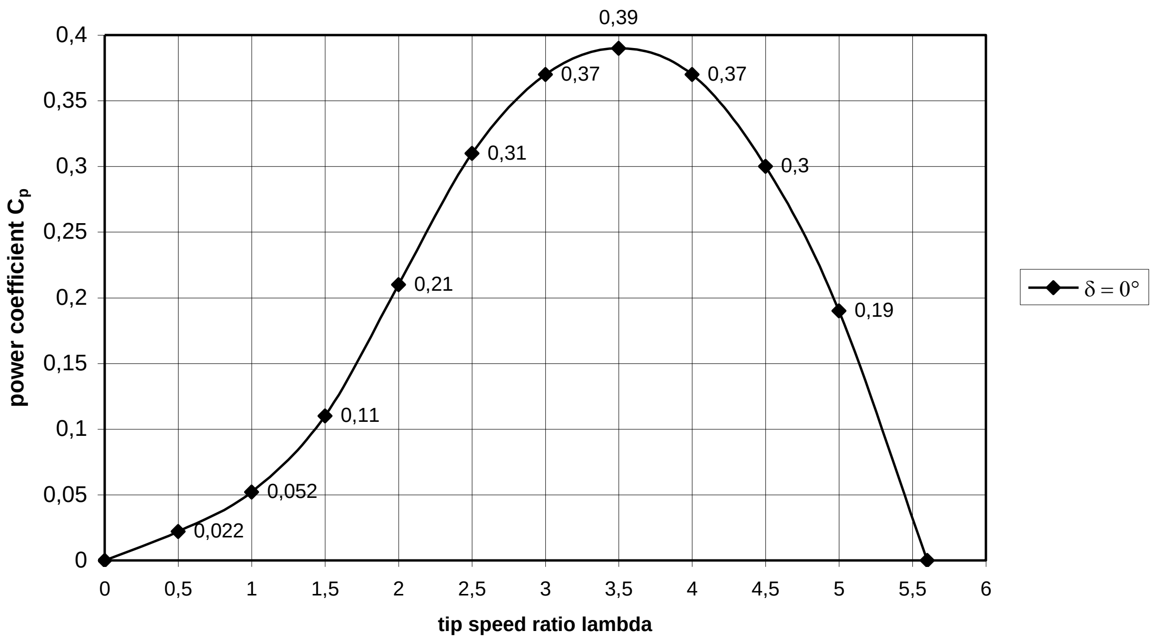

fig. 1 Estimated C_p-λ curve for the VIRYA-1.04 rotor for the wind direction perpendicular to the rotor ($\delta = 0°$)

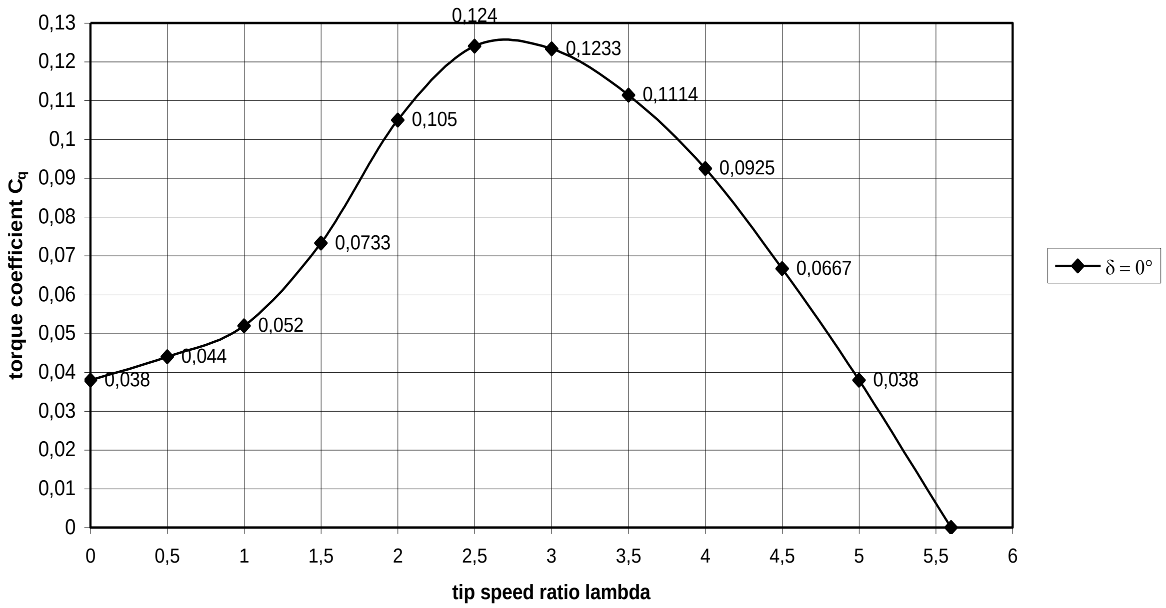

fig. 2 Estimated C_q-λ curve for the VIRYA-1.04 rotor for the wind direction perpendicular to the rotor ($\delta = 0°$)

5 Determination of the P-n curves, the optimum cubic line and the P_{el}-V curve

The determination of the P-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_p-λ curve of the rotor and the δ-V curve of the safety system together with the formulas for the power P and the rotational speed n. The C_p-λ curve is given in figure 1. The δ-V curve for a 1.5 mm aluminium is estimated on the basis of the proven δ-V curves of the VIRYA-1.8 and 2.2S windmills which have a 1 mm stainless steel vane blade and which have a rated wind speed of about 11 m/s. The estimated δ-V curve for a 1.5 mm aluminium vane blade is given in figure 3.

The head starts to turn away at a wind speed of about 5 m/s. For wind speeds above 8 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The P-n curve for 8 m/s will therefore also be valid for wind speeds higher than 8 m/s.

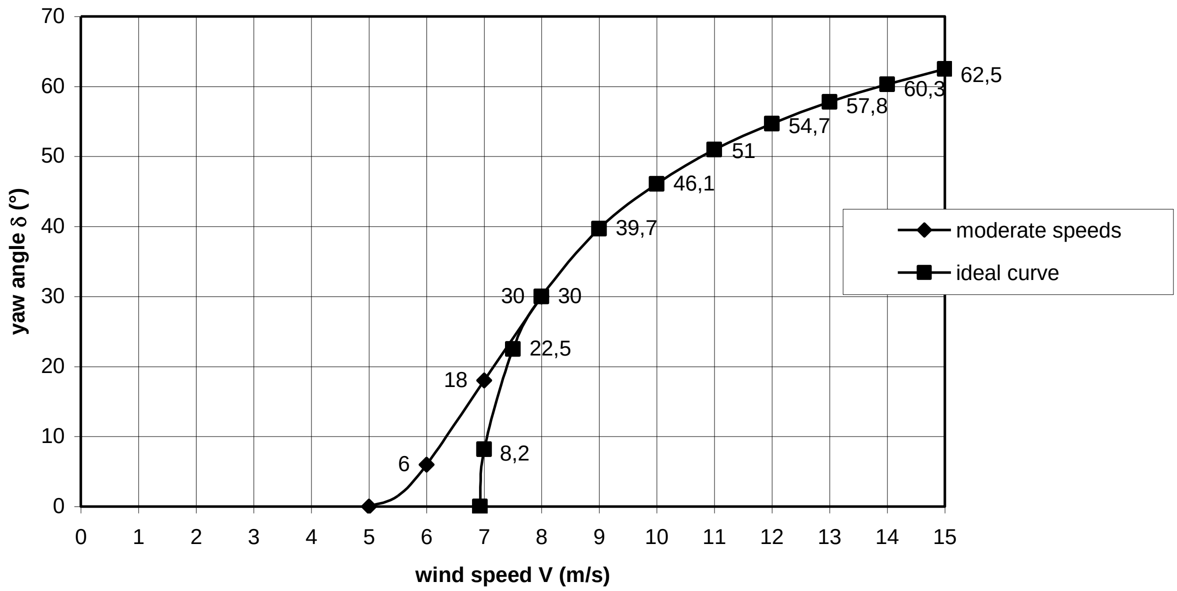

fig. 3 Estimated δ-V curve VIRYA-1.04 for a 1.5 mm aluminium vane blade

The P-n curves are used to check the matching with the P_{mech}-n curve of the generator for a certain gear ratio i (the VIRYA-1 has no gearing so i = 1). Because the P-n curve for low values of λ appears to lie very close to each other, the P-n curves are not determined for very low values of λ. The P-n curves are determined for C_p values belonging to λ is 2, 2.5, 3, 3.5, 4, 4.5, 5 and 5.6 (see figure 1). The P-n curves are determined for wind the speeds 2, 3, 4, 5, 6, 7 and 8 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for P and n are used which are given in chapter 7 of KD 35.

Substitution of R = 0.52 m in formula 7.1 of KD 35 gives:

$$n = 18.364 * \lambda * \cos\delta * V \qquad \text{(rpm)} \qquad (8)$$

Substitution of $\rho = 1.2$ kg / m^3 en R = 0.52 m in formula 7.10 of KD 35 gives:

$$P = 0.5097 * C_p * \cos^3\delta * V^3 \qquad \text{(W)} \qquad (9)$$

For a certain wind speed, for instance V = 3 m/s, related values of C_p and λ are substituted in formula 8 and 9 and this gives the P-n curve for that wind speed.

		V = 2 m/s $\delta = 0°$		V = 3 m/s $\delta = 0°$		V = 4 m/s $\delta = 0°$		V = 5 m/s $\delta = 0°$		V = 6 m/s $\delta = 6°$		V = 7 m/s $\delta = 18°$		V = 8 m/s $\delta = 30°$	
λ	C_p	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)
2	0.21	73.5	0.86	110.2	2.89	146.9	6.85	183.6	13.38	219.2	22.74	244.5	31.58	254.5	35.60
2.5	0.31	91.8	1.26	137.7	4.27	183.6	10.11	229.6	19.75	274.0	33.57	305.6	46.62	318.1	52.55
3	0.37	110.2	1.51	165.3	5.09	220.4	12.07	275.5	23.57	328.7	40.07	366.8	55.65	381.7	62.72
3.5	0.39	128.5	1.59	192.8	5.37	257.1	12.72	321.4	24.85	383.5	42.24	427.9	58.65	445.3	66.11
4	0.37	146.9	1.51	220.4	5.09	293.8	12.07	367.3	23.57	438.3	40.07	489.0	55.65	508.9	62.72
4.5	0.3	165.3	1.22	247.9	4.13	330.6	9.79	413.2	19.11	493.1	32.49	550.2	45.12	572.5	50.85
5	0.19	183.6	0.77	275.5	2.61	367.3	6.20	459.1	12.11	547.9	20.58	611.3	28.57	636.1	32.21
5.6	0	205.7	0	308.5	0	411.4	0	514.2	0	613.7	0	684.6	0	712.5	0

table 2 Calculated values of n and P as a function of λ and V for the VIRYA-1.04 rotor

The calculated values for n and P are plotted in figure 4. The optimum cubic line which is going through the tops of the P_{mech}-n curves is also given in figure 4.

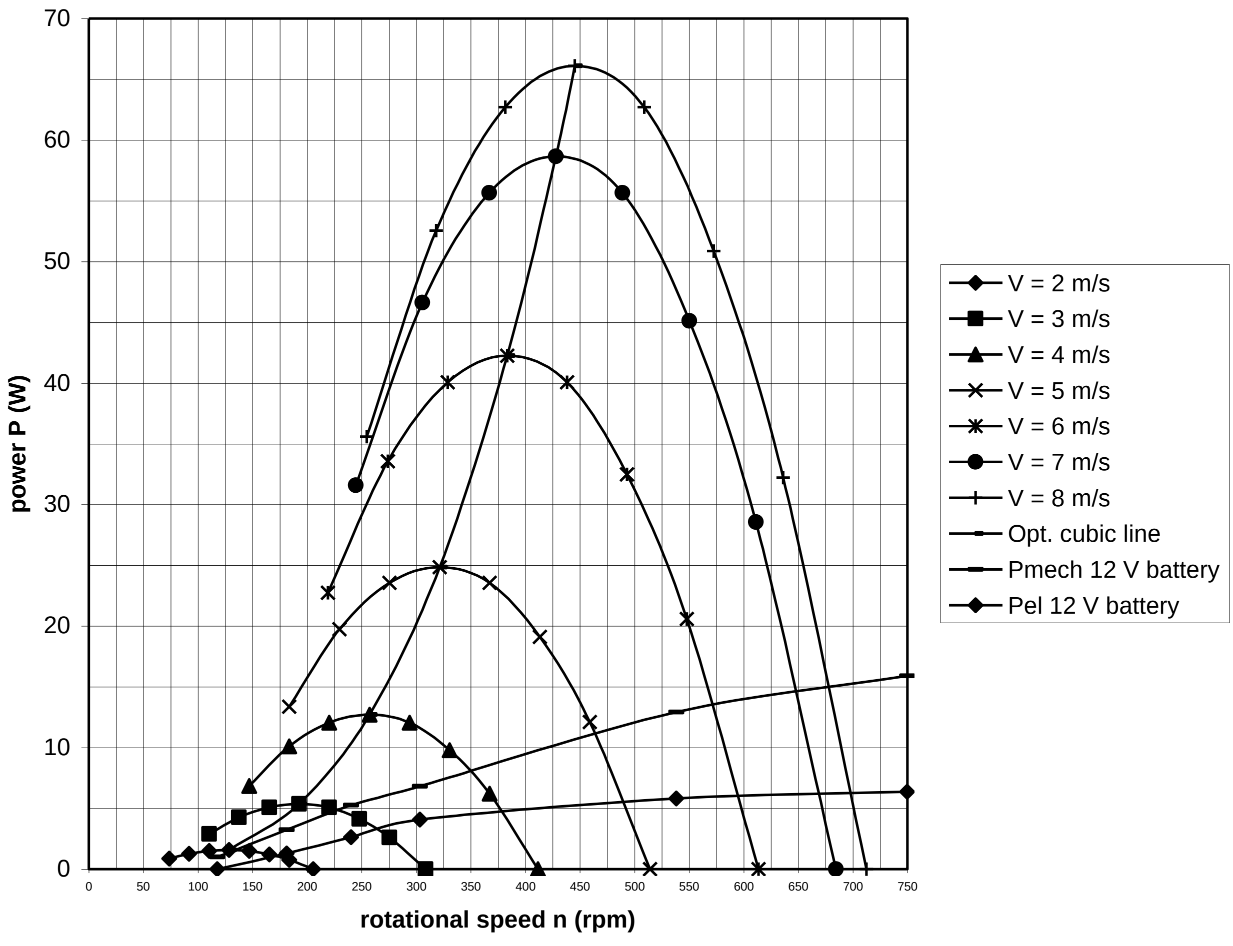

fig. 4 P-n curves and optimum cubic line of the VIRYA-1.04 rotor

The Nexus hub dynamo has been measured for an old 12 V car battery as load. The AC-current of the dynamo was rectified with a 1.5 A, 1-phase bridge rectifier. The dynamo was driven by a PM-DC motor which could run on variable speed. The dynamo was coupled to the motor hub by a flat belt. The rotational speed of the dynamo was measured by a laser rpm meter. The DC-voltage U was measured by a digital volt meter. The DC-current I was measured by an analogue volt meter. The electrical power P_{el} is the product of U * I.

The torque could not be measured and the P_{mech}-n curve is determined for an estimated efficiency curve which has a maximum of η = 0.6 for about n = 300 rpm. The estimated P_{mech}-n curve and the measured P_{el}-n curve for 12 V battery charging are also given in figure 4.

The working point for a certain wind speed is the point of intersection of the P_{mech}-n curve of the generator and the P-n curve of the rotor for that wind speed. The corresponding electrical power P_{el} is found by going down vertically from the working point up to the point of intersection with the P_{el}-n curve of the generator. This is done for all wind speeds and the values of P_{el} found this way are given in the P_{el}-V curve of figure 5.

In figure 4 it can be seen that the matching in between rotor and generator is only good for very low wind speeds. For high wind speeds, the working point is lying far to the right side of the optimum cubic line which means that the rotor is running almost unloaded and that the C_p is very low. The rotational speed for a wind speed of 8 m/s is about 680 rpm and the tip speed ratio is about 5.3.

The hub dynamo has 28 poles and so 28 preference positions in one revolution. The dynamo has been measured up to a maximum rotational speed of about 750 rpm. The dynamo makes noise caused by the preference positions. The noise at 750 rpm has a high frequency and is rather loud. Therefore it is not advised to mount the VIRYA-1.04 windmill on the roof of a house as the vibration will probably be felt and heard inside.

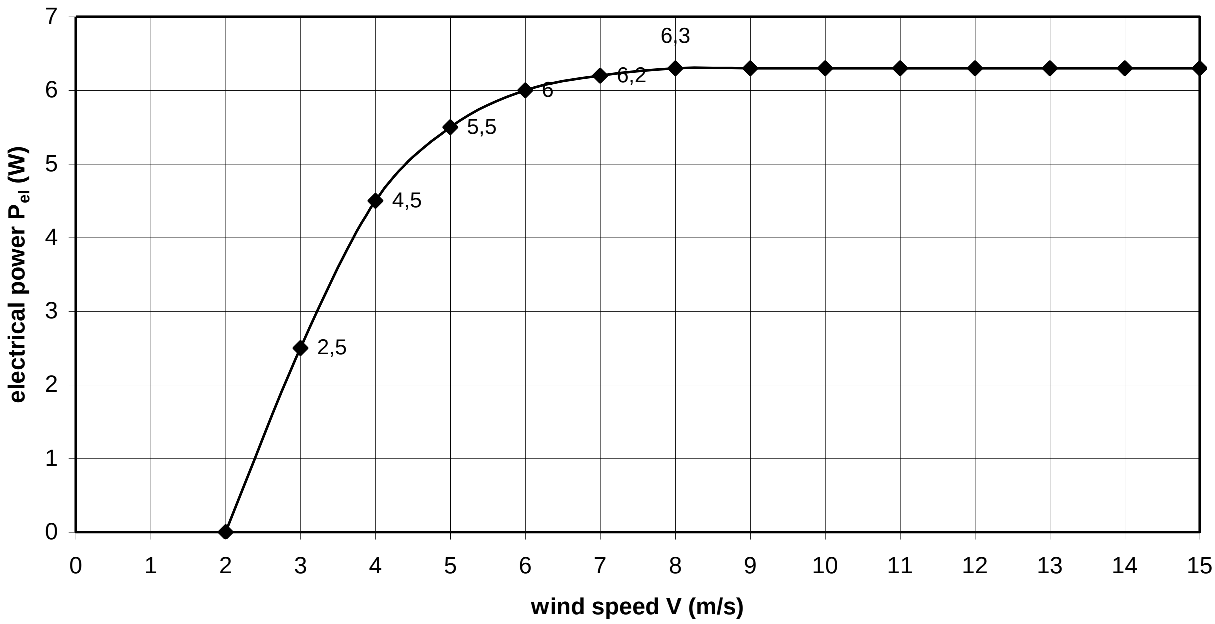

fig. 5 P_{el}-V curve of the VIRYA-1.04 windmill for 12 V battery charging

The wind speed where the generation of power starts is called the cut in wind speed $V_{cut\ in}$. In the P_{el}-V curve it can be seen that $V_{cut\ in}$ = 2 m/s. This is very low. In chapter 4 it was calculated that the effective starting wind speed is about 2.6 m/s. So there is some hysteresis in the P_{el}-V curve for 2 m/s < V < 2.6 m/s.

The maximum electrical power is about 6.3 W. However, an old car battery was used and the charging voltage at maximum power was about 14.6 V. The charging voltage for a new battery will be lower and therefore it is expected that the real maximum power will be about 6 W. A more accurate P_{el}-V curve can be determined if a new battery is used and if the dynamo is measured more accurately on a test rig with which it is also possible to measure the torque Q. However, the given P_{el}-V curve gives a good idea about the possibilities of using a hub dynamo for a very small wind turbine which is used for 12 V battery charging.

6 Calculation of the strength of the blades

The blades are flat just at the edge of the dynamo flange and the bending moment in the blade will be maximal at this position. The maximum torque level for short-circuit of a hub dynamo is certainly too low to use the generator as a brake to stop the rotor. So the rotor can only be stopped by turning it out of the wind. So normally the rotor will always rotate (except for very low wind speeds). The maximum thrust will be exerted on the rotor for the rated wind speed V = 8 m/s as it is assumed that the VIRYA-1.04 is turned out of the wind by the safety system, such that the thrust is constant for wind speeds higher than 8 m/s.

A blade has a width b = 125 mm and a height h = 1.5 mm. The blade root lies at 20 mm from the dynamo centre. The inner 30 mm of a blade is flat. The transition part, which has a length of 70 mm, transfers from flat to the cambered part at station E. The camber will increase as one is closer at station E. So also the moment of resistance of the blade will increase, but close to the flat part it will be rather low. Next it is assumed that the increase of the moment of resistance can be neglected for the first 20 mm of the transition part. The flange has an outer diameter of about 92 mm so the radius r = 46 mm. So it is assumed that the blade bends over the flat part in between r = 46 mm and r = 70 mm.

The blade is loaded by a bending moment with axial direction which is caused by the rotor thrust. The blade is also loaded by a gyroscopic moment. Because the flat part of the blade is rather thin it makes the blade connection elastic and therefore the blade will bend backwards at a low load. As a result of this bending, a moment with direction forwards is created by a component of the centrifugal force in the blade. The bending is substantially decreased by this moment and this has a favourable influence on the bending stress.

The rotor thrust for a yawing rotor is given by formula 7.4 of KD 35. The rotor thrust is the axial load of all blades together and exerts in the hart of the rotor. The thrust per blade $F_{t\,\delta\,bl}$ is the rotor thrust $F_{t\,\delta}$ divided by the number of blades B. This gives:

$$F_{t\,\delta\,bl} = C_t * \cos^2\delta * ½\rho V^2 * \pi R^2 / B \qquad \text{(N)} \qquad (10)$$

For the rotor theory it is assumed that every small area dA which is swept by the rotor, supplies the same amount of energy and that the generated energy is maximised. For this situation the wind speed in the rotor plane has to be slowed down till 2/3 of the undisturbed wind speed V. This results in a pressure drop over the rotor plane which is the same for every value of r. It can be proven that this results in a triangular axial load which forms the thrust and in a constant radial load which supplies the torque. The theoretical thrust coefficient C_t for the whole rotor is 8/9 = 0.889 for the optimal tip speed ratio. In practice C_t is lower because of the tip losses and because the blade is not effective up to the rotor centre. The effective blade length k’ of the VIRYA-1.04 rotor is only 0.35 m but the rotor radius R = 0.52 m. Therefore there is a disk in the centre with an area of about 0.107 of the rotor area on which almost no thrust is working. This results in a theoretical thrust coefficient C_t = 8/9 * 0.893 = 0.794. Because of the tip losses, the real C_t value is substantially lower. Assume this results in a real practical value of C_t = 0.75. It is assumed that the thrust coefficient is constant for values of λ in between λ_d and $\lambda_{unloaded}$.
Substitution of C_t = 0.75, δ = 30°, ρ = 1.2 kg/m^3, V = 8 m/s, R = 0.52 m and B = 3 in formula 10 gives $F_{t\,\delta\,bl}$ = 6.1 N.

For a pure triangular load, the same moment is exerted in the hart of the rotor as for a point load which exerts in the centre of gravity of the triangle. The centre of gravity is lying at 2/3 R = 0.347 m. Because the effective blade length is only k’, there is no triangular load working on the blade but a load with the shape of a trapezium as the triangular load over the part R – k’ falls off. The centre of gravity of the trapezium has been determined graphically and is lying at about r_1 = 0.37 m.

The maximum bending stress is not caused at the hart of the rotor but at the edge of the hub because the strip bends backwards from this edge. This edge is lying at r_2 = 0.046 m. At this edge we find a bending moment $M_{b\,t}$ caused by the thrust which is given by:

$M_{b\,t} = F_{t\,bl} * (r_1 - r_2)$ (Nm) (11)

Substitution of $F_{t\,bl}$ = 6.1 N, r_1 = 0.37 m and r_2 = 0.046 m gives $M_{b\,t}$ = 1.98 Nm = 1980 Nmm.

For the stress we use the unit N/mm^2 so the bending moment has to be given in Nmm. The bending stress σ_b is given by:

$\sigma_b = M / W$ (N/mm^2) (12)

The moment of resistance W of a strip is given by:

$W = 1/6\ bh^2$ (mm^3) (13)

(12) + (13) gives:

$\sigma_b = 6\ M / bh^2$ (N/mm^2) (M in Nmm) (14)

Substitution of M = 1980 Nmm, b = 125 mm and h = 1.5 mm in formula 14 gives σ_b = 42 N/mm^2. For this stress the effect of the stress reduction by bending forwards of the blade caused by the centrifugal force in the blade has not yet been taken into account. Next it is investigated how far the strip bends backwards as a result of the thrust load and what influence this bending has on the centrifugal moment. Hereby it is assumed that the cambered part of the blade is not bending and that the blade is bending in between the hub and the point r_3 = 0.07 m = 70 mm. So the length of the strip l which is loaded by bending is given by:

$l = r_3 - r_2$ (mm) (15)

The load from the blade on the strip at r_3 can be replaced by a moment M and a point load F. F is equal to $F_{t\,\delta\,bl}$. M is given by:

$M = F * (r_1 - r_3)$ (Nmm) (16)

The bending angle ϕ (in radians) at r_3 for a strip with a length l is given by (combination of the standard formulas for a moment plus a point load):

$\phi = l * (M + ½\ Fl) / EI$ (rad) (17)

The bending moment of inertia I of a strip is given by:

$I = 1/12\ bh^3$ (mm^4) (18)

(15) + (16) + (17) + (18) gives:

$\phi = 12 * F * (r_3 - r_2) * \{(r_1 - r_3) + ½\ (r_3 - r_2)\} / (E * bh^3)$ (rad) (19)

Substitution of F = 6.1 N, r_3 = 70 mm, r_2 = 46 mm, r_1 = 370 mm, E_{al} = 0.7 * 10^5 N/mm^2, b = 125 mm and h = 1.5 mm in formula 19 gives: ϕ = 0.0186 rad = 1.06°.

This is an angle which can not be neglected. In report R409D (ref. 4) a formula is derived for the angle ε with which the blade moves backwards if it is connected to the hub by a hinge. This formula is valid if both the axial load and the centrifugal load are triangular. For the VIRYA-1.04 this is not exactly the case but the formula gives a good approximation. The formula is given by:

$$\varepsilon = \arcsin\left(\frac{C_t * \rho * R^2 * \pi}{B * A_{pr} * \rho_{pr} * \lambda^2}\right) \quad (°) \qquad (20)$$

In this formula A_{pr} is the cross sectional area of the airfoil (in m^2) and ρ_{pr} is the density of the used airfoil material (in kg/m^3). For a plate width of 125 mm and a plate thickness of 1.5 mm it is found that $A_{pr} = 0.000188\ m^2$. The blade is made of aluminium sheet with a density ρ_{pr} of about $\rho_{pr} = 2.7 * 10^3\ kg/m^3$. At high wind speeds the hub dynamo will give only a limited load and the rotor will run almost unloaded. It is assumed that the rotor is loaded such that it runs at a tip speed ratio $\lambda = 5.3$. Substitution of $C_t = 0.75$, $\rho = 1.2\ kg/m^3$, $R = 0.52$ m, $B = 3$, $A_{pr} = 0.000188\ m^2$, $\rho_{pr} = 2.7 * 10^3\ kg/m^3$ and $\lambda = 5.3$ in formula 20 gives: $\varepsilon = 1.02°$.

This angle is smaller than the calculated angle of 1.06° with which the blade would bend backwards if the compensating effect of the centrifugal moment is not taken into account. This means that the real bending angle will be less than 1.02°.

The real bending angle ε is determined a follows. A thrust moment $M_t = 1.98$ Nm is working backwards and M_t is independent of ε for small values of ε. A bending moment M_b is working forwards and M_b is proportional with ε. $M_b = 1.98$ Nm for $\varepsilon = 1.06°$. A centrifugal moment M_c is working forwards and M_c is also proportional with ε. $M_c = 1.98$ Nm for $\varepsilon = 1.02°$. The path of these three moments is given in figure 6. The sum total of $M_b + M_c$ is determined and the line $M_b + M_c$ is also given in figure 6.

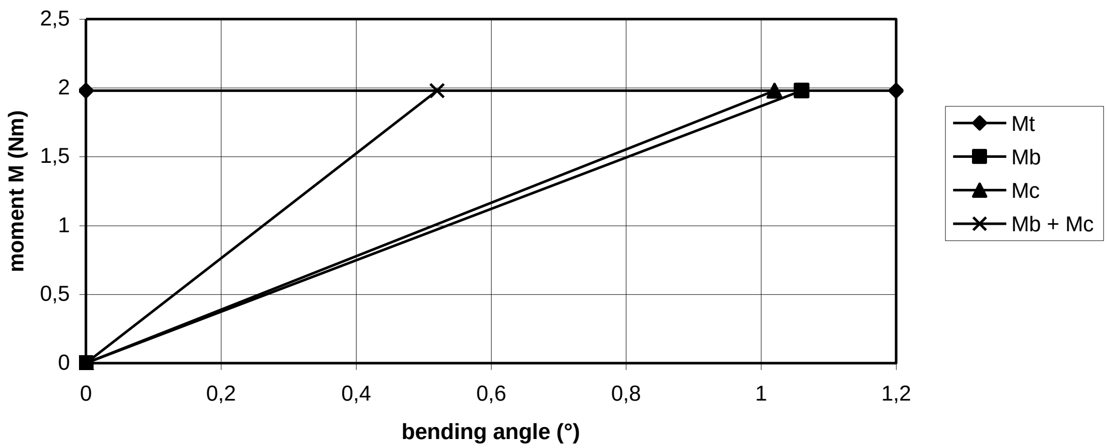

fig. 6 Path of M_t, M_b, M_c, and $M_b + M_c$ as a function of ε

The point of intersection of the line of M_t with the line of $M_b + M_c$ gives the final angle ε. In figure 6 it can be seen that $\varepsilon = 0.52°$. This is a factor 0.49 of the calculated angle of 1.06°. Because the bending stress is proportional to the bending angle it will also be a factor 0.49 of the calculated stress of 42 N/mm^2 resulting in a stress of about 21 N/mm^2. This is a rather low stress. This is a low stress but up to now the gyroscopic moment, which can be rather large, has not yet been taken into account.

The gyroscopic moment is caused by simultaneous rotation of the rotor and the head. One can distinguish the gyroscopic moment in a blade and the gyroscopic moment which is exerted by the whole rotor on the rotor shaft and so on the head. On a rotating mass element dm at a radius r, a gyroscopic force dF is working which is maximum if the blade is vertical and zero if the blade is horizontal and which varies with sinα with respect to a rotating axis frame. α is the angle with the blade axis and the horizon. So it is valid that $dF = dF_{max} * \sin\alpha$. The direction of dF depends on the direction of rotation of both axis and dF is working forwards or backwards. The moment dF * r which is exerted by this force with respect to the blade is therefore varying sinusoidal too. However, if the moment is determined with respect to a fixed axis frame it can be proven that it varies with $dF_{max} * r \sin^2\alpha$ with respect to the horizontal x-axis and with $dF_{max} * \sin\alpha * \cos\alpha$ with respect to the vertical y-axis. For two and more bladed rotors it can be proven that the resulting moment of all mass elements around the y-axis is zero.

For a single blade and for two bladed rotors, the resulting moment of all mass elements with respect to the x-axis is varying with $\sin^2\alpha$, so just the same as for a single mass element. However, for three and more bladed rotors, the resulting moment of all mass elements with respect to the x-axis is constant. The resulting moment with respect to the x-axis for a three (or more) bladed rotor is given by the formula:

$$M_{gyr\ x\text{-}as} = I_{rot} * \Omega_{rot} * \Omega_{head} \qquad \text{(Nm)} \qquad (21)$$

In this formula I_{rot} is the mass moment of inertia of the whole rotor, Ω_{rot} is the angular velocity of the rotor and Ω_{head} is the angular velocity of the head. The resulting moment is constant for a three bladed rotor because adding three $\sin^2\alpha$ functions which make an angle of 120° which each other, appear to result in a constant value. The three functions are given in figure 7. It can be proven for a three bladed rotor that the sum value of the three blades is equal to 3/2 of the peak value of one blade.

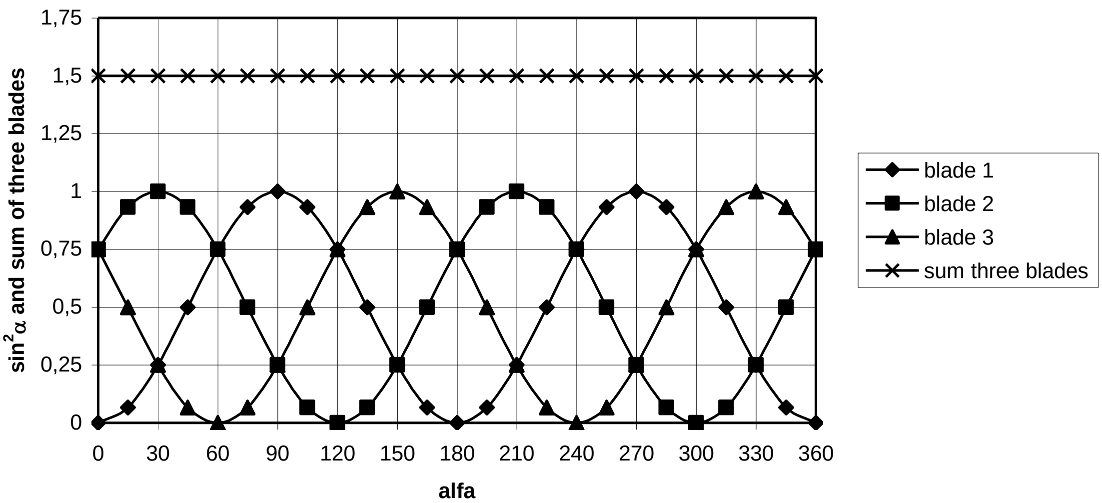

fig. 7 Path of $\sin^2\alpha$ and the sum of three blades

For the calculation of the blade strength we are not interested in the variation of the gyroscopic moment with respect to a fixed axis frame but in variation of the moment in the blade itself so with respect to a rotation axis frame for which it was explained earlier that the moment is varying sinusoidal. If the blade is vertical both axis frames coincide and the moment for both axis frames is the same. The maximum moment in one blade is then 2/3 of the sum moment as given by formula 21.

The variation of the moment in the blade with respect to a rotating axis frame is therefore given by:

$M_{gyr\ bl} = 2/3 \sin\alpha * I_{rot} * \Omega_{rot} * \Omega_{head}$ (Nm) (22)

For a three bladed rotor, the moment of inertia of the whole rotor I_{rot} is three times the moment of inertia of one blade I_{bl}. Therefore it is valid that:

$M_{gyr\ bl} = 2 \sin\alpha * I_{bl} * \Omega_{rot} * \Omega_{head}$ (Nm) (23)

Up to now it is assumed that the blades have an infinitive stiffness. However, in reality the blades are flexible and will bend by the fluctuations of the gyroscopic moment. Therefore the blade will not follow the curve for which formula 22 and 23 are valid. I am not able to describe this effect physically but the practical result of it is that the strong fluctuation on the $\sin^2\alpha$ function is rather flattened. However, the average moment is assumed to stay the same as given by formula 21. I estimate that the flattened peak value is given by:

$M_{gyr\ bl\ max} = 1.2 * I_{bl} * \Omega_{rot} * \Omega_{head}$ (Nm) (24)

For the chosen blade geometry it is calculated that I_{bl} = 0.024 kgm^2. The maximum loaded rotational speed of the rotor can be read in figure 4 for V = 8 m/s and it is found that n_{max} = 680 rpm. This gives $\Omega_{rot\ max}$ = 71.2 rad/s (because $\Omega = \pi * n / 30$).

It is not easy to determine the maximum yawing speed. The VIRYA-1.04 is provided with the hinged side vane safety system which has a light vane blade and a large moment of inertia of the whole head around the tower axis. This is because the vane arm is a part of the head. For sudden variations in wind speed and wind direction the vane blade will therefore react very fast but the head will follow only slowly. It is assumed that the maximum angular velocity of the head can be 0.4 rad/s at very high wind speeds.
Substitution of I_{bl} = 0.024 kgm^2, $\Omega_{rot\ max}$ = 71.2 rad/s en $\Omega_{head\ max}$ = 0.4 rad/s in formula 24 gives: $M_{gyr\ bl\ max}$ = 0.82 Nm = 820 Nmm.
Substitution of M = 820 Nmm, b = 125 mm and h = 11.5 mm in formula 14 gives $\sigma_{b\ max}$ = 18 N/mm^2. This value has to be added to the bending stress of 21 N/mm^2 which was the result of the thrust because there is always a position were both moments are strengthening each other. This gives $\sigma_{b\ tot\ max}$ = 39 N/mm^2. The minimum stress is 21 - 18 = 3 N/mm^2. So the stress is not becoming negative and therefore it is not necessary to take the load as a fatigue load.

For the blade half hard aluminium (MCB quality AW-5754 (Al Mg3) has been chosen. The 0.2 % deformation limit depends on the hardness of the material (in between H12 and H22) but the minimum value is given as 130 N/mm^2. However, this is the stress for a pulling force. The 0.2 % deformation limit for a bending moment is much higher and it is assumed that it is about 180 N/mm^2. The allowable bending stress for a non fatigue load will be lower than the 0.2 % deformation limit and it is expected that a bending stress of 140 N/mm^2 is allowed for a non fatigue load. The load is a non fatigue load as the stress is not becoming negative for the assumed maximum yawing speed. The maximum calculated stress is 39 N/mm^2, so an aluminium blade with a thickness of 1.5 mm is strong enough and even has a large reserve.

7 References

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Appendix 1 Sketch VIRYA-1.04 rotor, $\lambda_d = 3.5$, B = 3