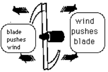
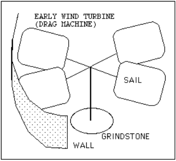
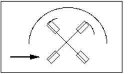
Wind turbine rotor blades take power from the wind by <u>slowing it down.</u>

This is done by applying a force to the wind, and the wind applies that same force to the blades.





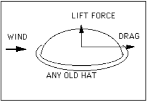
Objects in the path of a stream of air experience a 'downwind' force called <u>drag.</u>

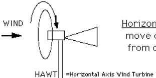


The drag force was used by the earliest wind turbines. It is easy to understand how this force causes the blades to turn, but such rotors are <u>very slow</u> and the blades which are moving <u>upwind</u> actually slow the rotor down.

Drag is the force of wind pushing straight downwind.

But there is another force called <u>'lift'</u> which always works at right angles to the wind direction.





<u>Horizontal axis</u> wind turbine blades never move downwind, so they can get no help from drag forces. Instead they use lift.

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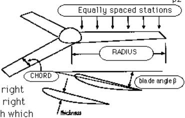
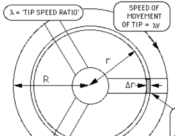
http://www.windmission.dk/workshop/books.html

To create a <u>blade design</u> we need to specify the chord width and blade setting angle β at each of a series of <u>stations</u> along the span of the blade.

At each station we will create the right shape of the blade to produce the right loading (lift) for the 'bit of wind' with which it will have to deal.





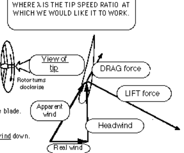
The process of calculating the best loading and thence the best shape is known as '<u>finite element analysis</u>', and it looks at what each bit of the blade needs to do.

THE BIT OF THE BLADE AT RADIUS r SWEEPS A FRACTION OF THE TOTAL SWEPT AREA, AND HAS THE JOB OF SLOWING THIS BIT OF WIND DOWN BY THE RIGHT AMOUNT TO SATISFY THE BETZ CRITERION.

THE AREA OF WIND IT SWEEPS WILL BE $2\pi r \Delta r$.
ITS HEADWIND WILL BE $(r/R)\lambda V$
WHERE λ IS THE TIP SPEED RATIO AT WHICH WE WOULD LIKE IT TO WORK.

The <u>apparent</u> wind which a blade 'sees' is altered by its own speed through the air.

This <u>headwind</u> adds to the real wind to give the apparent wind, which creates the lift and drag forces.



The headwind <u>rotates</u> the direction of the forces on the blade.
The <u>Drag</u> force opposes the blade's movement.
The <u>Lift</u> force assists the blade's movement.
Both forces also push the blade downwind and slow the <u>wind</u> down.

The mathematics of lift and drag.

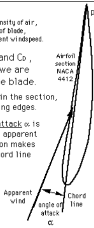
$$\text{LIFT} = C_L(\rho/2)AV_a^2$$
$$\text{DRAG} = C_D(\rho/2)AV_a^2$$

where ρ is the density of air, A is the area of blade, and V_a is the apparent windspeed.

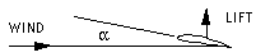
Lift and Drag forces depend on the <u>Coefficients</u> C_L and C_D, which in turn depend on the cross <u>section</u> of blade we are using, and on the <u>angle</u> α at which the wind strikes the blade.

The <u>chord line</u> is the longest line in the section, joining the leading and trailing edges.

The <u>angle of attack</u> α is the angle the apparent wind direction makes with the chord line



WE ARE MORE ACCUSTOMED TO LOOKING AT THE WINGS OF AIRCRAFT, WHICH ARE THIS WAY AROUND:

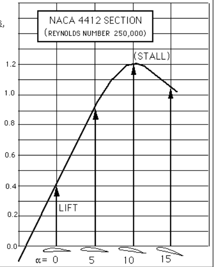


You cannot <u>calculate</u> the lift and drag coefficients.

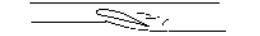
They are measured experimentally in <u>wind tunnels</u>, and recorded in books.

Here is a typical graph of <u>lift vs. angle of attack α</u>.

As α increases, so does the lift, until a point is reached where the blade <u>stalls.</u>



AIR FLOW SEPARATES FROM THE BACK OF THE BLADE IN STALL.



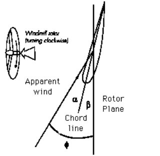
LIFT FAILS AND DRAG INCREASES RAPIDLY.

Most flattish objects will give a similar sort of LIFT/α curve. But cambered, steamlined sections yeild better <u>lift/drag</u>.

When designing a wind turbine rotor, the <u>angle α</u> will depend on the angle of the apparent wind φ, and the blade angle β.

So we have control over α, and thus control over the lift and drag produced by the blade.

We shall need to optimise the lift force, to satisfy the Betz criterion, but the blade will not work well unless the <u>drag</u> is minimised.
So we have to choose a section and an angle of attack, where the <u>lift/drag ratio</u> is high.

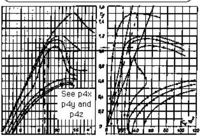


$$\beta = \phi - \alpha$$

Finding the exact best angle α can be an involved process, because the lift and drag coefficients depend on both the <u>section</u> and the <u>Reynolds number</u> (a measure of the size and speed of the blade).

THE REYNOLDS NUMBER IS 68500 X CHORD (M) X APPARENT WIND SPEED (M/S)

IF D=2m AND λ=5 AND V=5m/s THEN RENOLDS NO. IS ABOUT 120,000

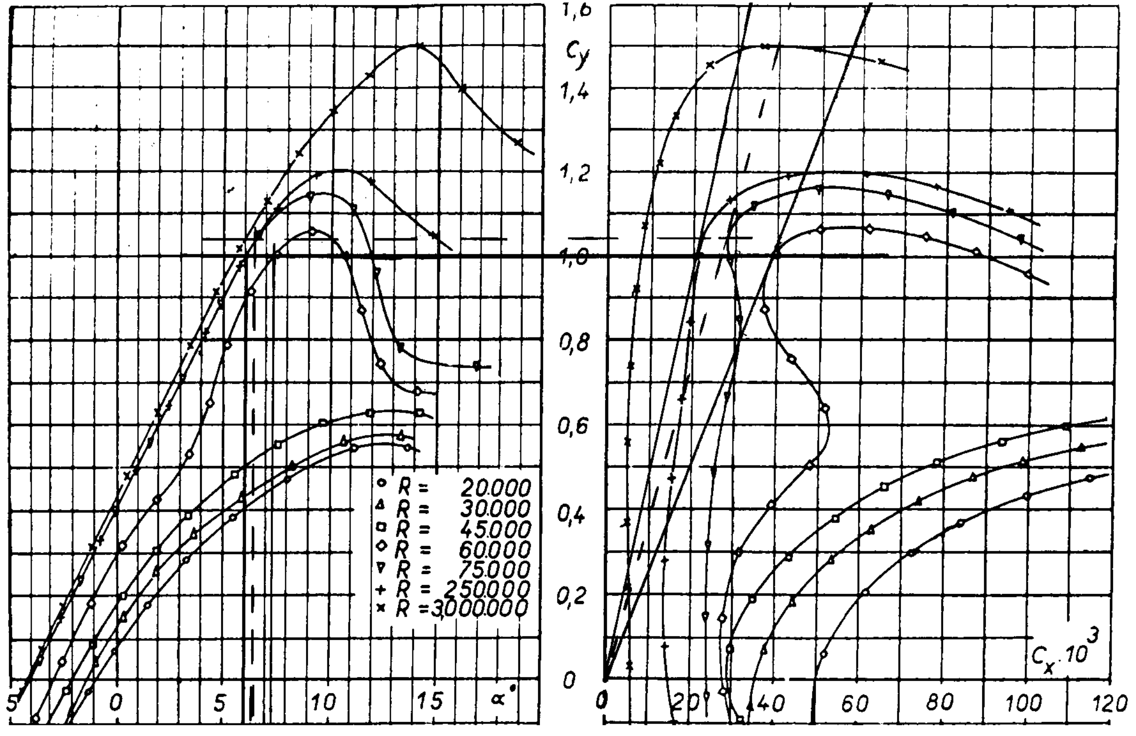


On the left is a pair of graphs which again relate to the NACA 4412 section for several different Reynolds numbers.
The lefthand graphs shows <u>lift/α</u>.
The righthand one shows <u>lift/drag</u>.
The straight lines through zero, represent particular lift/drag ratios.
Best lift/drag ratio for a given Reynolds number occurs where the lift/drag line is rotated as far as possible anticlockwise, so that it just touches the curve as a tangent.
For the NACA 4412, this point of contact is where CL is about 1, and α is about 6.

Note that low Reynolds number leads to poor lift and low lift/drag ratio, which can pose problems for rotors with narrow chord widths in low winds. There are other sections (eg 'ClarkY' and 'K2') which have better performance than the NACA4412 at low Reynolds number.

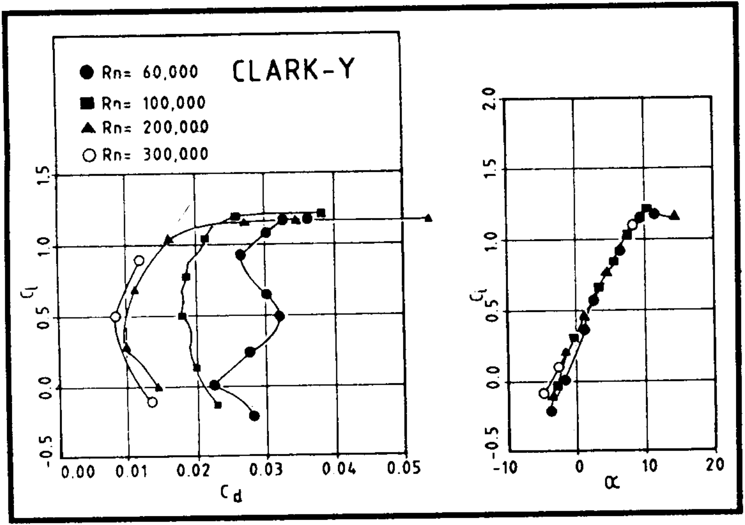
In practice, most sections will produce their best lift/drag at an angle of attack around 5 degrees, so <u>as a general rule</u>, where detailed data is not available, we can say that the blade angle β should be set to give this angle of attack, thus:

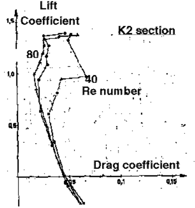
$$\beta = \phi - 5$$



1,6
C_y
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0
5
0
5
10
15
α°
0
20
40
60
80
100
120
$C_x \cdot 10^3$
○ R = 20.000
△ R = 30.000
□ R = 45.000
◇ R = 60.000
▽ R = 75.000
+ R = 250.000
× R = 3.000.000

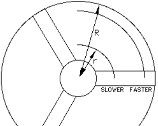
WIND TUNNEL TEST POLARS



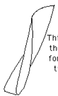


Lift
Coefficient
K2 section
80
40
Re number
Drag coefficient

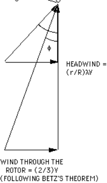
To specify blade angle β we need to know the angle φ at which the apparent wind strikes the rotor plane.



Headwind is greater near the tip (where r=R) than it is near the root, so the angle φ changes.



This means that the ideal shape for the blade is twisted, like this.



CALCULATING THE CORRECT BLADE SETTING ANGLE β

$\beta = \phi - \alpha$

WHERE $\tan(\phi) = (2V/3)/(r/R)\lambda V = 2R/(3r\lambda)$

SO THE BLADE ANGLE β IS

$\beta = \mathrm{ATAN}(2R/3r\lambda) - \alpha$

WHERE α IS USUALLY AROUND 5 DEGREES.

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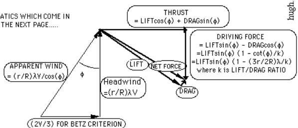
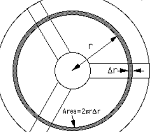
MORE MATHEMATICS WHICH COME IN USEFUL ON THE NEXT PAGE.....

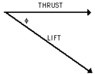




Having worked out β we still need to work out the <u>Chord width</u>. Here is the logic:

Each blade element has a certain <u>band</u> of wind to process.
As radius r grows smaller near the centre, the amount of wind in the band gets smaller too.

The outer parts of the blade therefore do the most work. The inner part is less important but needs a different shape.



To satisfy Betz, the wind in each part of the swept area of the rotor must be slowed down to 1/3 of its upstream velocity, and this slowing is done by the THRUST force, which is very closely related to the LIFT force.

NEGLECTING DRAG (very small error), THRUST = LIFT $\cos(\phi)$

FOR BETZ, THRUST = $(4/9)\rho A V^2 = (4/9)\rho(2\pi r\Delta r)V^2$

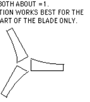
AND WE KNOW THAT LIFT = $CL(\rho/2)BC\Delta r(\text{APPARENT WIND})^2$
$= CL(\rho/2)BC\Delta r(\lambda V(r/R)/\cos(\Phi))^2$

where B is the number of blades,
CL is the lift coefficient,
C is the chord width,
at radius r,
and V is the free windspeed.
BCΔr is the area of blade used to produce lift at radius r.

THIS LEADS TO A ROUGH EXPRESSION FOR THE <u>CHORD WIDTH C</u> WHICH WILL PRODUCE THE RIGHT AMOUNT OF THRUST TO MEET THE <u>BETZ CONDITION</u>

$$C = \frac{16\pi R\ (R/r)}{9\lambda^2 B}$$

WARNING: FOR SIMPLICITY, WE HAVE ASSUMED THAT CL AND COS(φ) ARE BOTH ABOUT =1.
THIS EQUATION WORKS BEST FOR THE OUTER PART OF THE BLADE ONLY.



<u>CONCLUSIONS</u>

C IS INVERSELY PROPORTIONAL TO <u>RADIUS</u> r.
so the blade shape should be tapered
C IS INVERSELY PROPORTIONAL TO BLADE <u>NUMBER</u> B
so fewer blades will be wider blades
C IS INVERSELY PROPORTIONAL TO <u>TIP SPEED RATIO</u> **SQUARED**
so doubling speed means cutting blade width down to 1/4

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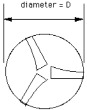
Back of envelope blade design:-



1. Choose rotor diameter D to suit your power requirements

Diameter **(m)**	(Watts) Power
1	50-100
2	250-500
3	500-1000
4	1000-2000
5	2000-3000

2. Choose a tip speed ratio λ.

You are free to use is trial and error here. I suggest you opt for a tip speed ratio between 5 and 8.

Tip speed ratio will affect rpm.
shaft speed=60λV/(πD) rpm

IF YOU HAVE A GENERATOR WITH KNOWN POWER OUTPUT AND KNOWN RPM, AND YOU WANT TO BUILD A WINDMILL TO FIT THAT, THEN YOU MAY FIND THIS FORMULA USEFUL:

DIAMETER = (POWER(47λ/RPM)^3)^0.2
('^0.2' MEANS THE FIFTH ROOT)

FOR EXAMPLE IF POWER = 500 W
AND RPM = 300 RPM
AND CHOSEN TIP SPEED RATIO = 5
THEN BEST DIAMETER WILL BE

DIAMETER = (500x(47x5/300)^3)^0.2
= (500x(0.783)^3)^0.2
=240^0.2=3metres

3. Decide how many blades B to use

(B=3 is the best.
Or try $B=80/\lambda^2$)

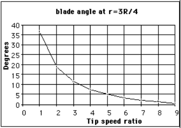
4. The width of the blade C in the outer portion, will be :

$$C = 4D/(\lambda^2 B)$$

For example if D=2m, and tip speed ratio = 7 and B=2, then C = 4x2/49x2= 0.08m (or 8cm).

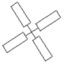
The outer part is the most important, but the inner part should be made wider, to help with starting torque.

5. To find the best blade setting angle β, read it from this graph:-



THIS IS BASED ON THE IDEAL ANGLE FOR A POINT NEAR THE TIP.

STRAIGHT, UNTAPERED, UNTWISTED BLADES
IN PRACTICE MANY WIND TURBINE BLADES ARE BUILT WITH CONSTANT WIDTH AND CONSTANT BLADE ANGLE, LIKE THIS. THERE IS SURPRISINGLY LITTLE LOSS OF EFFICIENCY BY MAKING THIS COMPROMISE.



BUT THERE ARE OTHER GOOD REASONS TO USE A TWIST AND A TAPER:

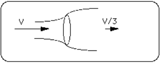
BETTER STARTING

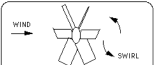
STRONGER BLADE ROOT

Factors affecting the power coefficient

(Where the lost energy goes)

Loss 1 is the wind which escapes around the side of the rotor. Betz figures out that the best we can do is catch 0.593 of the power, and that to catch even that much we need to slow the wind down to 1/3 of its upstream, free velocity V.





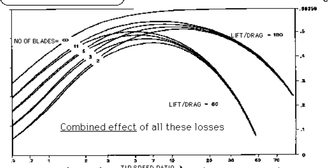
Loss 2 is the lost power in the swirl created by high torque rotors. Glauert figured out that this is worst at low tip speed ratios.

Loss 3 is due to the fact that we are not able to be everywhere at once. Where there are only a small number of blades, the thrust loading is higher, and some wind prefers to go around the tips. This is known as 'Tip Loss'

DRIVING FORCE
= LIFTsin(φ) (1 - (3r/2R)λ/k)
where k is LIFT/DRAG RATIO (see p5)

SO LIFT/DRAG MUST INCREASE WITH INCREASING TIP SPEED RATIO OR DRAG TAKES A HEAVY TOLL.

Loss 5 is drag loss, which depends on LIFT/DRAG ratio. It gets worse for high tip-speed-ratio rotors, where the lift force is rotated furthest from the direction of blade movement.



(CHART BY HERR DOERNER)

http://www.ifb.uni-stuttgart.de/~doerner/edesignphil.html

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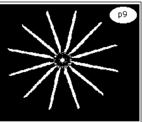
So what is the <u>best design</u> for a wind turbine rotor?

From the graphs, it looks as if a tip speed ratio around 5 is ideal, with as many blades as possible. The trouble with having lots of blades is that they have to be very narrow, or run at very low tip speed ratio (or both), to satisfy the Betz condition.

The perfect wind turbine rotor has an infinite number of infinitely narrow blades.



Source : Windmission

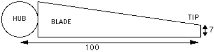
The 'windflower' type of rotor (right), created by Claus Nybroe at Windmission, follows this logic.

HERE IS A 12-BLADED 'WINDFLOWER' ROTOR DESIGNED FOR TIP SPEED RATIO λ= 3.6. ARGUABLY THIS IS THE MOST EFFICIENT SHAPE OF ROTOR.

IN PRACTICE THIS APPROACH IS RARELY USED BECAUSE THE ROTOR IS TOO SLOW. AT HIGHER TIP SPEED RATIOS, 3 BLADES WORK BETTER, IN SPITE OF THE LOSSES.

Due to the low Re-numbers the blade profile must be carefully selected and rather thin. To obtain strength and torsional stiffness, this requires a composite structure and skilled workmanship.

Here is a less ambitious planform shape for a blade:



IF $C = \dfrac{16\pi R\,(R/r)}{9\lambda^2 B}$

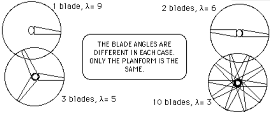
THEN $B = \dfrac{16\pi R\,(R/r)}{9\lambda^2 C}$

AT THE TIP, C=(7/100)R, SO

$B = \dfrac{80}{\lambda^2}$

RULE OF THUMB ONLY FOR THE BLADE DEPICTED

Once you have chosen a blade planform, then the number of blades is dictated by the <u>tip speed ratio λ</u>:-



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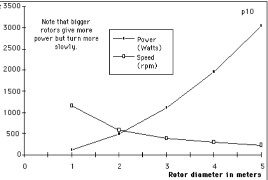
High speed blades
(pros and cons)

The graph to the right shows the speeds and electrical power outputs of windmills with a range of rotor sizes, running at tip speed ratio of 5, in a 12m/s rated windspeed.

For this graph, power is calculated on the basis of rotor Cp=0.25 and other losses=40% overall, which is easily possible for small wind turbines. (Other losses are friction, iron, copper and rectifier losses to produce the electricity output.)

Good machines will exceed this performance.



Choice of rotor size (diameter) depends on power required.

Choice of tip speed ratio λ depends on many factors. High tip speed ratio results in higher shaft speed is more efficient for generating electricity, which often outweighs these disadvantages:-

1. Noise from the blades is higher
2. Vibration in case of 2-bladed (or 1-bladed).
3. Blades edges, at high air-speeds suffer erosion.
4. Reduced rotor efficiency, due to drag, and tip loss.
5 Starting difficulties, if the shaft is stiff to turn.

STARTING TORQUE CAN BE ESTIMATED FROM THE FORMULA

$$\text{TORQUE} = \frac{V^2 R^3}{(\text{DESIGN TIP SPEED RATIO})^2}$$

FOR EXAMPLE A 2m DIAMETER WITH TIPS SPEED RATIO λ = 5 ROTOR IN A 4m/s WINDSPEED WILL HAVE STARTING TORQUE

$$\text{TORQUE} = \frac{4^2 1^3}{5^2} = 0.64 \text{ Nm}$$

N.B. THIS IS ONLY AN APPROXIMATION!

BLADE TIPS TRAVELLING AT SPEEDS IN EXCESS OF 80m/s WILL SUFFER FROM EROSION OF THE LEADING EDGES DUE TO IMPACT OF SMALL PARTICLES BORN BY THE WIND. THIS CAN BE COUNTERED TO SOME DEGREE, BY THE USE OF SPECIAL TOUGH COATINGS.

A ROTOR WITH TIP SPEED RATIO 7 IN A 12m/s WIND OR A 5m DIAMETER ROTOR RUNNING AT 350rpm WILL BE AT RISK FROM BLADE EROSION.

THE EFFECT INCREASES DRAMATICALLY WITH INCREASING SPEED